MAGLIB

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MAGLIB

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INTRODUCTION

The MAGLIB software results from our forty-year experience in mission analysis and geophysics software for magnetospheric projects as well as personal research on charged particle motion and quantitative magnetic field modeling. In the early times, (Araks I & II, 1972), the main demand was a correct description of the near Earth magnetic field. We used the external magnetic field model of Roederer and Sauer based on two dipoles to obtain the conjugacy in the northern hemisphere for the rockets launched in the Kerguelen islands. The results were very rough but showed both the diurnal and the seasonal motion of the conjugate points. The same model was used for Geos I. The partial failure of the launch of Geos I by a Mc Donnell Douglas Thor Delta rocket led W.P. Olson of the same Company to offer us an advanced model as a compensation in 1974. This model was used for mission analysis and projects like Geos II, Sambo, Arcade3 and Viking (French experiments). The version in our hands was limited to distances less than 15 Re in the night side but gave reasonable excursions of the conjugate points for a-synchronous spacecraft. All these low-altitude spacecraft (we consider the synchronous altitude as near-Earth compared to the size of the Magnetosphere) were not very demanding in terms of software: calculation of the geomagnetic local time, calculation of the Mc Ilwain L, etc. With Interball and its auroral and eccentric probes, a great step was accomplished. It was necessary to use more accurate external magnetic field models with a realistic topology of the field lines in the tail region. This was achieved with the Tsyganenko models (1982, 1987 and 1989). These models were incorporated in the software. It was also necessary to take into account the crossing of the Magnetopause and to avoid any calculation involving the magnetic field outside the Magnetosphere. The eccentric Interball spacecraft with a 200000. km apogee crossed different magnetospheric regions, radiation belts, aurora oval, polar cap, Magnetosheath, Neutral Sheet: adequate routines were developed to define the boundaries of these regions. This software paved the way to the development of an interactive calculation and visualization tool, the OCGM, which also incorporated sophisticated orbit extrapolation. A further step was accomplished for the Cluster project with the calculation of the distances to the different boundaries, spacecraft speed in various coordinate systems and a variable Solar Wind velocity inducing a variable subsolar distance. In between we finally clarified the mysterious corrected geomagnetic coordinates (mysterious only for the low latitude Space physicist!) and added the calculation of the eccentered-tilted dipole. Unfortunately all the mysteries have not been solved. Among these, the absence of a renewed B, L coordinates system, the present one being defined for an internal geomagnetic field of the early sixties. Also the comparison of observations labeled in geophysical coordinates of same L and MLT separated by a fifteen-year interval. For Geos I we were very active finding some reasonable magnetospheric model. Later studies (Kosik, 1983) showed that during quiet times a tilted dipole proved sufficient, the ring current giving axisymmetric results for the synchronous altitude. The continuous advance in the knowledge of the Magnetosphere as well as the improvement of the models will lead to further modifications of this library, not to mention future interplanetary missions to Jupiter or Io. For the immediate needs the library contains about 160 routines. The routines have been divided into 9 sections. The first section contains the initialization routines to be called before any geophysical calculation or after a time update. The coordinate transformations form the second chapter. The regions and boundaries are described in the

third chapter. The section on models comprises internal and external magnetic field models. In this section all the DGRF internal field models since 1945 have been included.
To avoid errors the internal magnetic field is calculated with two types of routines where the coefficients are stored differently. The comparison of the results thus guarantees the correct use of the coefficients. For the external models, model Kosik 99 has been added, its field line topology being a good compromise between the fast Tsyganenko 87, 89 models and the more precise but time consuming Tsyganenko 96. The calculation of the geomagnetic local time and of the Mc Ilwain L parameter are in the geophysics calculations chapter. The calculations of the geomagnetic local time or the tilt angle require the knowledge of the Sun position and of the Greenwich Meridian with respect to an inertial coordinate system. These calculations and related subjects form the core of the astronomical and celestial mechanics chapter. Basic mathematical routines such as matrix multiplication or determination of roots are in the mathematics chapter. Time and date calculations, transformations between julian and gregorian dates are collected in one chapter, the date calculations. Finally to avoid unnecessary headaches caused by stupid results whose origin lie in a wrong choice of parameters a series of control routines have been developed and form the control routines chapter. The whole set of routines represents 24000 lines of documented fortran77. The software was carefully tested and set to quality standards by Michel Lagreca and Suzanne Le Guillou from CS-SI. In a separate volume a series of user routines give practical examples of calculations by combining the elementary bricks of the library. Finally a third volume will contain the physics and the corresponding mathematical equations. In 2000 an HTML version should be available offering a hypertext search of the type of calculation in its three aspects: the physics and the mathematics, the basic routine and the example. This work has benefited from the interaction with numerous Space physicists involved both in the projects and fundamental research. We particularly acknowledge Roger Gendrin for his encouragement and support during two decades.
In this chapter we review the different coordinate systems in use in magnetospheric physics: geocentric inertial, geocentric, solar ecliptic, geomagnetic or dipolar, solar magnetic, solar magnetospheric and the aberrated or Solar Wind coordinate system.

Version 2.0
In this new version, the coefficients of the internal magnetic field models have been updated and the IGRF 2005 introduced.
A paragraph has been added about the applications of the Galperin L parameter and its connection to quantitative mathematical models.

Version 2.1
In version 2.1 the "Tables of Internal Magnetic Field Coefficients" paragraph has been rearranged for the sake of readability. The fact that the most recent set of coefficients is provisory is also clearly indicated
In paragraph 7.4 “The Galperin L Parameter”, it has been added a reference to a publication by J.C. Kosik

Version 3.0
In this new version, the coefficients of the internal magnetic field models have been updated and the IGRF 2010 introduced.

Version 4.0

In this new version, the coefficients of the internal magnetic field models have been updated and the IGRF-12 model for 2015 introduced.

1. COORDINATE TRANSFORMATIONS

1.1 THE GEOCENTRIC INERTIAL COORDINATE SYSTEM

This coordinate system is defined in the following way:

- The origin O is at the barycenter of the Earth.
- ZI axis is along the Celestial Poles axis.
- XI axis is in the Equator and points towards the Vernal Point defined as the intersection of the Ecliptic and Equator great circles.
- YI axis is defined through the cross product ZI x XI.

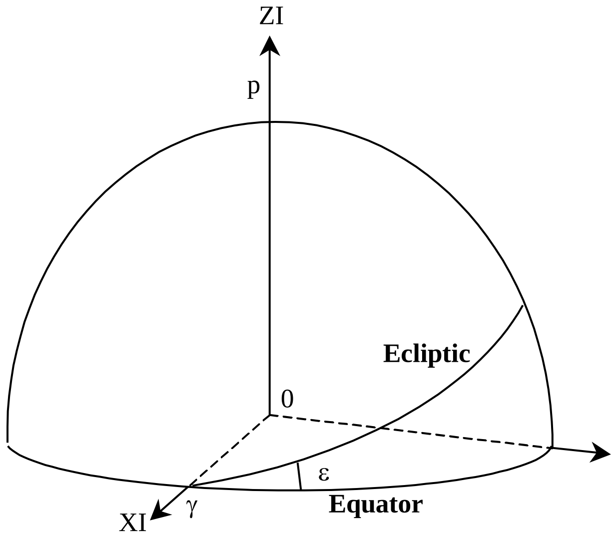

Figure 1

1.2 THE GEOCENTRIC TERRESTRIAL SYSTEM

The geocentric coordinate system is not inertial and rotates with the Earth. It is defined in the following way:

- The origin O is at the center of the Earth.
- ZG axis is along the Celestial Poles axis.
- XG axis is in the Equator and the plane OXZ contains the Greenwich Meridian.
- YG axis is defined through the cross product ZG x XG.

The Greenwich Meridian is defined by the right ascension of Greenwich. The right ascension is counted from the Vernal Point and positive eastwards.

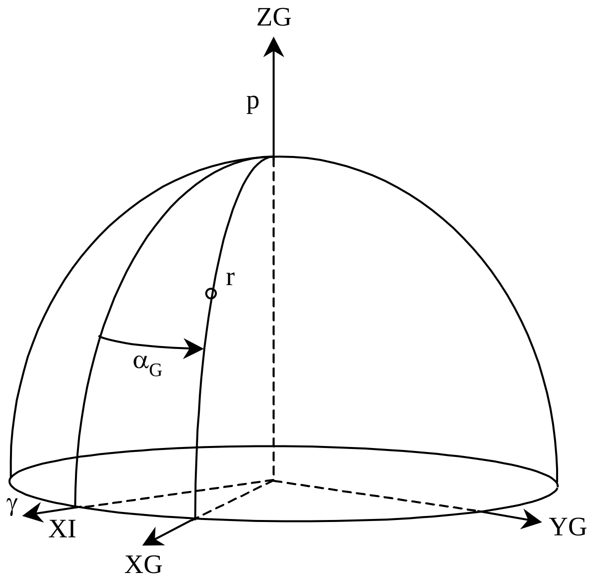

Figure 2

The transformation matrix from the geocentric inertial frame of reference to the geocentric frame of reference is:

$$RIG = \begin{bmatrix} \cos\alpha_G & \sin\alpha_G & 0 \\ -\sin\alpha_G & \cos\alpha_G & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The transpose matrix RGI transforms the geocentric components of a vector into the geocentric inertial components. Matrices RIG (R for rotation, I for inertial, G for geocentric) and RGI are obtained with routine **roig**. The transformation of the coordinates and velocity components of a spacecraft from the inertial coordinate system to the geocentric coordinate system is performed in **pvig.**

1.3 THE SOLAR ECLIPTIC COORDINATE SYSTEM

In the Solar Ecliptic coordinate system the ZSE axis is along the Ecliptic Pole direction Q and the XSE axis points towards the Sun. The angle between ZSE and ZI axes is the obliquity of the Ecliptic with respect to the Equator. The YSE axis is defined by the cross, product ZSE x XSE. The position of the Sun along the Ecliptic is defined by its longitude L. The Sun longitude is calculated in the routine SUN. The transformation of the components of a vector from the geocentric inertial system into the solar ecliptic system GSE is obtained by the products of two transformations: a rotation around the XI inertial axis by ε, then a rotation around the ZSE axis by an angle L:

$$[M_{IE}] = [M_L][M_\varepsilon]$$

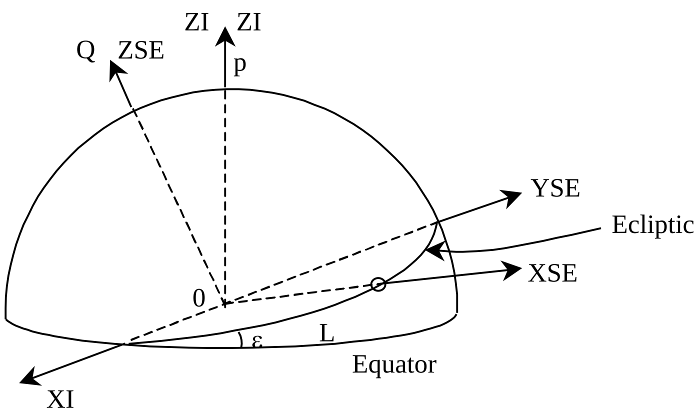

Figure 3

The matrices are defined as follows:

$$[M_\varepsilon]=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\varepsilon & \sin\varepsilon\\ 0 & -\sin\varepsilon & \cos\varepsilon\end{bmatrix} \qquad [M_L]=\begin{bmatrix}\cos L & \sin L & 0\\ -\sin L & \cos L & 0\\ 0 & 0 & 1\end{bmatrix}$$

The product of these two matrices gives matrix RISE (R for rotation, I for inertial, SE for solar ecliptic).

$$[RISE]=\begin{bmatrix}\cos L & \cos\varepsilon\sin L & \sin\varepsilon\sin L\\ -\sin L & \cos\varepsilon\cos L & \sin\varepsilon\cos L\\ 0 & -\sin\varepsilon & \cos\varepsilon\end{bmatrix}$$

The matrix transformation from the solar ecliptic frame of reference to the inertial frame of reference is RSEI, and the transpose matrix is RISE. These matrices are calculated in the subroutine **roise** (contraction of rotation from inertial to solar ecliptic).
To transform a vector from the geocentric into the solar scliptic coordinate system it is necessary to apply the transformation from geocentric to inertial than the transformation from inertial to solar ecliptic. The transformation matrix is called RGSE.

$$[RGSE]=[RISE][RGI]$$

$$[RGSE]=\begin{bmatrix}\cos\alpha_G\cos L+\sin\alpha_G\cos\varepsilon\sin L & -\sin\alpha_G\cos L+\cos\alpha_G\cos\varepsilon\sin L & \sin\varepsilon\sin L\\ -\cos\alpha_G\sin L+\sin\alpha_G\cos\varepsilon\cos L & \sin\alpha_G\sin L+\cos\alpha_G\cos\varepsilon\cos L & \sin\varepsilon\cos L\\ -\sin\alpha_G\sin\varepsilon & -\sin\varepsilon\cos\alpha_G & \cos\varepsilon\end{bmatrix}$$

The matrix RSEG transforms a vector from the solar ecliptic coordinate system into the geocentric coordinate system and is the transpose matrix of RGSE. These matrices are calculated in routine **rogse** (contraction of rotation from geocentric to solar-ecliptic). Routine **geose** transforms the geocentric components of a vector into solar-ecliptic components. Routine **segeo** performs the opposite transformation.

1.4 THE GEOMAGNETIC OR TILTED DIPOLE COORDINATE SYSTEM

The geomagnetic coordinate system is derived from the geocentric coordinate system by two transformations:

- A rotation of angle φ_d around the axis Zg in the anticlockwise sense which transforms the triedron OXgYgZg into the triedron OX1Y1Z1.
- A rotation of angle θ_d around axis Y1 in the southward direction which transforms OX1Y1Z1 into the final frame of reference OXdYdZd. The values of θ_d and φ_d are obtained with the first three harmonics of the Earth geomagnetic potential (cf. chapter on Internal Magnetic Field).

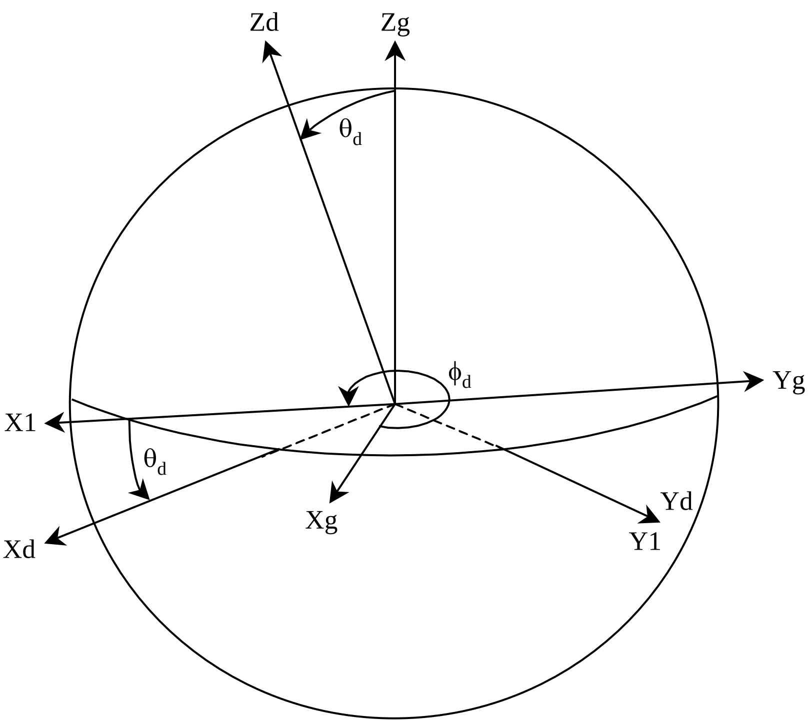

Figure 4

We obtain the transformation matrices:

$$[M_\varphi] = \begin{bmatrix} \cos\varphi_d & \sin\varphi_d & 0 \\ -\sin\varphi_d & \cos\varphi_d & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and
$$[M_\theta] = \begin{bmatrix} \cos\theta_d & 0 & -\sin\theta_d \\ 0 & 1 & 0 \\ \sin\theta_d & 0 & \cos\theta_d \end{bmatrix}$$

The final transformation matrix RGDIP is the product of these two matrices:

$$[RGDIP] = [M_\theta][M_\varphi]$$

$$[RGDIP] = \begin{bmatrix} \cos\theta_d \cos\varphi_d & \cos\theta_d \sin\varphi_d & -\sin\theta_d \\ -\sin\phi_d & \cos\varphi_d & 0 \\ \sin\theta_d \cos\varphi_d & \sin\theta_d \sin\varphi_d & \cos\theta_d \end{bmatrix}$$

The matrix RGDIP and its transpose RDIPG (matrix transformation from dipole to geocentric) are calculated in the routine **rogdip**. Routine **geodip** transforms the geocentric components of a vector into dipolar components. Routine **dipgeo** performs the opposite transformation.

1.5 THE SOLAR MAGNETIC COORDINATE SYSTEM

The solar magnetic coordinate system can be deduced from the geomagnetic coordinate system by a single rotation about Zd axis. The Xsm axis is located in the meridian plane which contains the Sun direction:

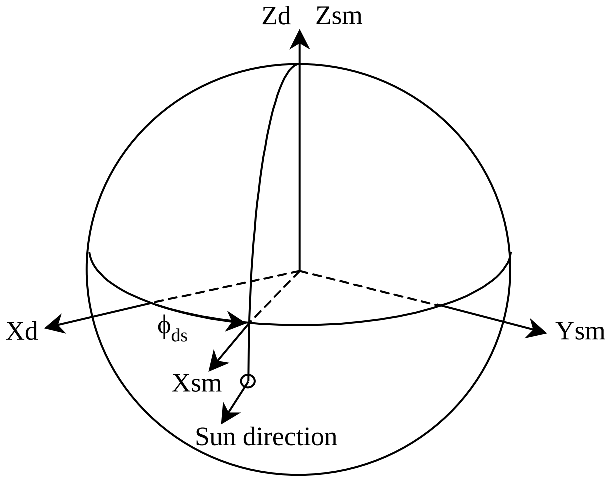

Figure 5

The calculation of the transformation matrix implies the knowledge of the geomagnetic longitude of the Sun. This geomagnetic longitude can be calculated from the position of the Sun in the geocentric frame of reference and using the transformation matrix RGDIP. The routine SUN gives the right ascension α_s and the declination δ_s of the Sun. With matrices RIG and RGDIR the geomagnetic coordinates X_{ds}, Y_{ds}, Z_{ds} of the Sun are obtained:

$$\begin{bmatrix} X_{ds} \\ Y_{ds} \\ Z_{ds} \end{bmatrix} = [RGDIP][RIG] \begin{bmatrix} \cos\delta_s \cos\alpha_s \\ \cos\delta_s \sin\alpha_s \\ \sin\delta_s \end{bmatrix}$$

From Xds, Yds, Zds we get the geomagnetic longitude φ_{ds} and the geomagnetic latitude λ_{ds} of the Sun. The angle λ_{ds} is also the tilt angle and called T. The transformation matrix RDSM from the dipole system to the solar magnetic system is simply:

$$[RDSM]=\begin{bmatrix}\cos\varphi_{ds} & \sin\varphi_{ds} & 0\\ -\sin\varphi_{ds} & \cos\varphi_{ds} & 0\\ 0 & 0 & 1\end{bmatrix}$$

Finally the transformation matrix RGSM from the geocentric coordinate system to the solar magnetic coordinate system is the product of two matrices:

$$[RGSM]=[RDSM][RGDIP]$$

The transformation matrix RGSM (Rotation from Geocentric to Solar Magnetic) and the transpose matrix RSMG (Rotation from Solar Magnetic to Geocentric) are calculated in the routine **rogsm**. Routine **geosm** transforms the geocentric components of a vector into solar magnetic components. Routine **smgeo** performs the opposite transformation.

1.6 THE SOLAR MAGNETOSPHERIC COORDINATES

The solar magnetospheric frame of reference is deduced from the solar magnetic coordinate system through a rotation about Ysm axis. Xgsm axis points towards the Sun. Ygsm axis is along Ysm axis. The Zgsm axis forms a right-handed frame of reference with the other two axes. The amount of rotation is defined by the angle U counted positive from Zsm towards Xsm (figure 6). The matrix transformation RSMGSM is defined simply as:

$$[RSMGSM]=\begin{bmatrix}\cos U & -\sin U & 0\\ \sin U & \cos U & 0\\ 0 & 0 & 1\end{bmatrix}$$

However U is opposite to the geomagnetic latitude of the Sun λ_{ds}. We apply the previous matrix and the relation U = - λ_{ds} in our software.

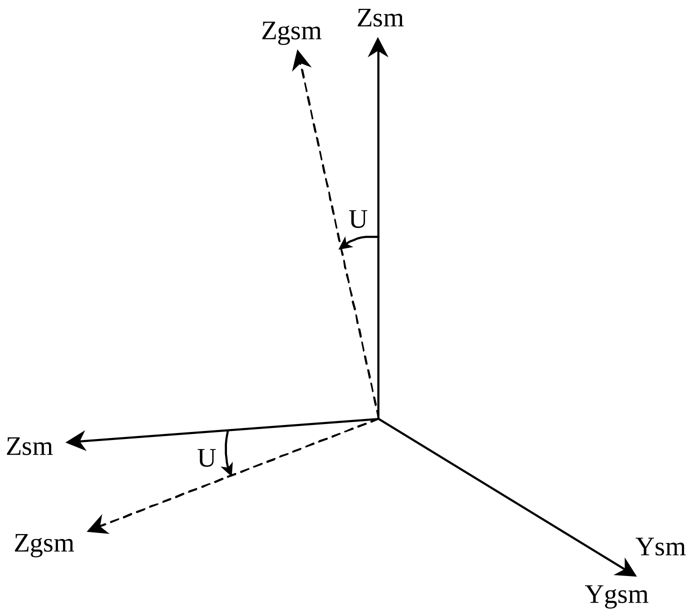

Figure 6

The matrix transformation RSMGSM and its transpose RGSMSM are calculated in the routine ROSMGS. The routine **geogsm** transforms the geocentric components of a vector into solar magnetospheric components. The routine **gsmgeo** performs the opposite transformation.

1.7 THE TILT ANGLE T

The TILT angle T is also the geomagnetic latitude of the Sun. The tilt angle is therefore negative around the Winter Solstice and positive around the Summer Solstice. The tilt angle is calculated in the routines **inigeo1** (Cluster), **inigeom** (Geolib), **inigeomv** (from 1945 to 2000).

1.8 THE SOLAR WIND COORDINATE SYSTEM

The geocentric Solar Wind coordinate system can be deduced from the geocentric solar ecliptic coordinate system through a rotation of angle A. This rotation takes into account the angle between the Solar Wind direction and the direction of the SUN. This angle also called ABERRATION originates in the orbital motion of the Earth around the Sun. In a frame of reference linked to the Earth a Solar Wind particle will be closer to the Z axis and have a tilted trajectory with respect to the Sun direction (figure 7). The angle A can be calculated. As the Solar Wind velocity, around 400 km/s is much greater than the Earth velocity, 30 km/s we have:

$$A = \frac{V_E}{V_{SW}}$$

which gives A around 4°.

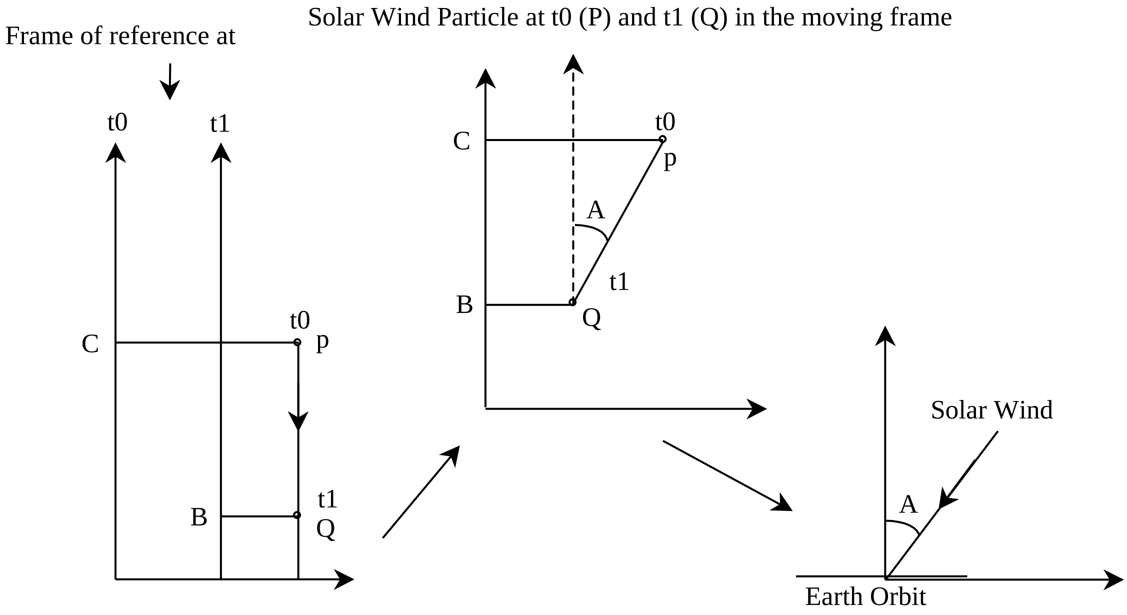

Figure 7

The geocentric Solar Wind coordinate system can be deduced from the geocentric solar ecliptic coordinate system through a rotation A:

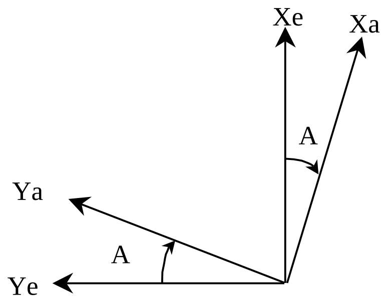

Figure 8

The matrix transformation from solar ecliptic components to Solar Wind components is:

$$\begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} = \begin{bmatrix} \cos A & \sin A & 0 \\ -\sin A & \cos A & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix} \qquad \text{where } A = -4°$$

The transformation of solar ecliptic components of a vector into Solar Wind components is performed in **aberrm.**

1.9 CLASSICAL VERIFICATIONS

Many errors can occur in the mathematical calculation of the matrices as well as in the software. It is not always obvious to find the errors but in some cases a frame of reference differs from another frame of reference by a rotation along one of the axes. As a consequence the components of the vector along this rotation axis must remain the same. From the previous paragraphs one must have:

Zd (component along the dipole)	=	Zsm (Z component in the solar magnetic frame)
Ygsm (solar magnetospheric)	=	Ysm (solar magnetic)
Za (aberrated solar ecliptic)	=	Ze (non aberrated solar ecliptic)

REFERENCES

Ref. 1 C. Russel: Geophysical Coordinates Transformations, Cosmic Electrodynamics, 1971, 2, 184-186, D. Reidel Publishing Company

2. BOUNDARIES AND REGIONS

2.1 INTRODUCTION

The Magnetosphere can be defined as the location where the Solar Wind interacts with the Earth's magnetic field. The Solar Wind is a plasma with an average velocity of 400 km/s which compresses the magnetic field into a cometary shape. The diameter of this comet is about 60 earth radii and the tail extends over several hundred earth radii.

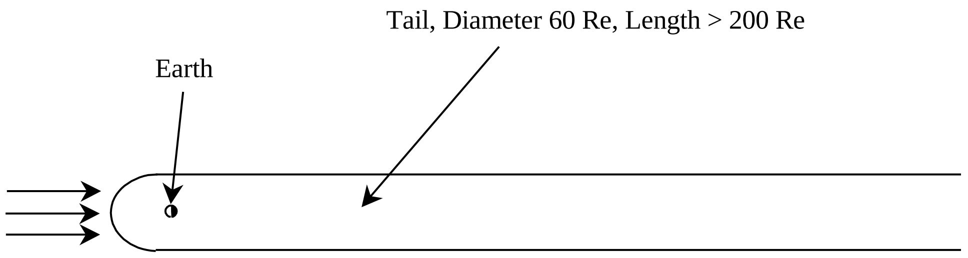

Figure 1

The distant regions are shown in the following figure.

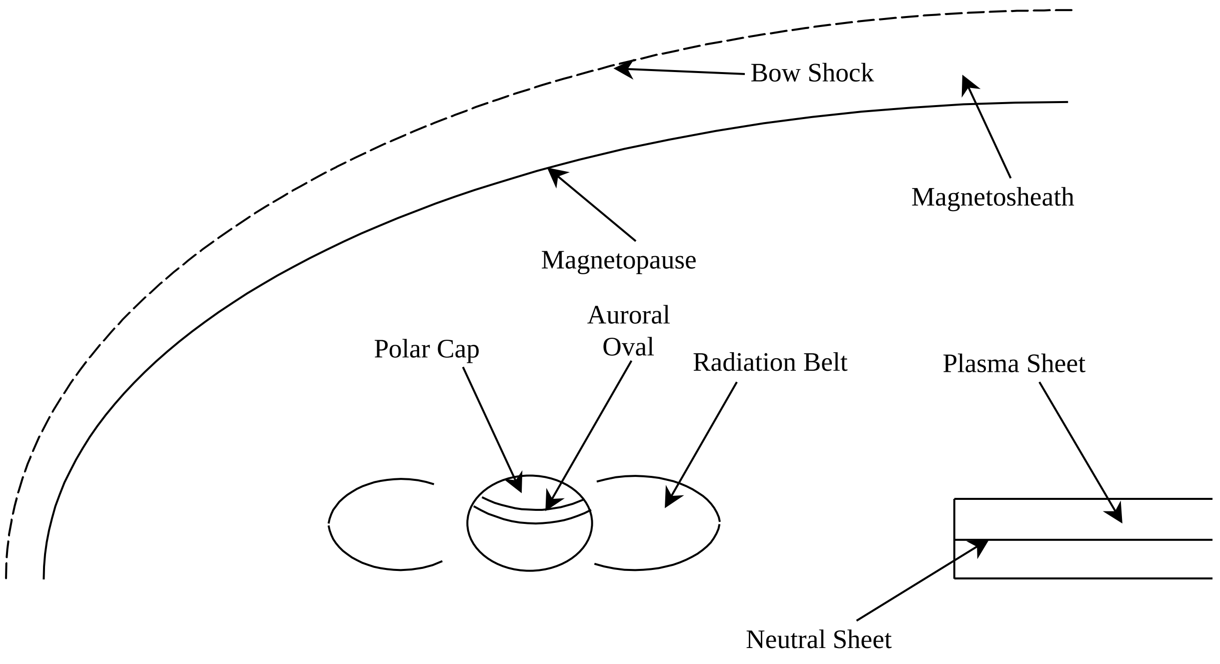

Figure 2

The subsolar point is approximately at 10 Re and is also the intersection of the Magnetopause with the Earth-Sun line. Along this same line the Bow-Shock is encountered at 15 Re. In a meridian plane perpendicular to the Sun Earth line the Magnetopause is encountered at 15 Re and the Bow-Shock is encountered at 20 Re.

Near the Earth we have several regions:

- The Radiation Belt which can be split into two belts: an Inner Belt bounded by a dipolar shell extending up to 3.5 Re in the equatorial plane and an Outer Belt bounded by a dipolar shell which extends up to 6. Re in the equatorial plane. The Inner Belt is the most "energetic" with protons of MeV energies.
- The Plasmasphere.
- The north and south Auroral Ovals where Auroras are observed.
- The Diffuse Auroral Region.
- The Cusp where magnetic field lines originating from the Magnetopause concentrate when they reach the Earth.

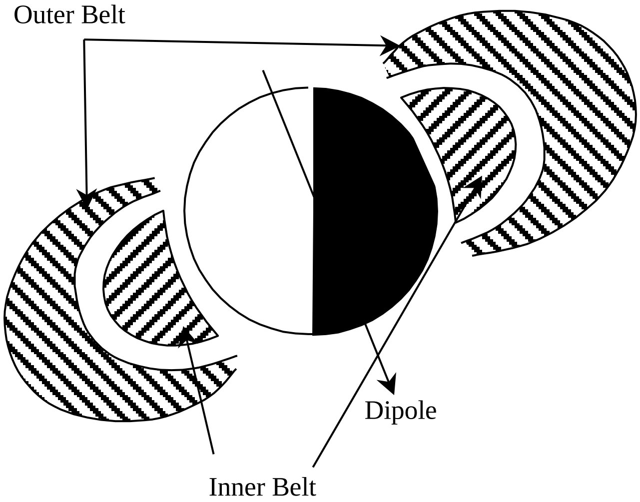

Figure 3

For the mathematical definition of the regions we have chosen two approaches:

- Near the Earth where the magnetic field is less sensitive to external perturbations the regions are defined by their L-shell boundaries
- In the outer regions where the magnetic field is sensitive to the Solar Wind conditions we have chosen a probabilistic approach. The Magnetopause and the Bow-Shock were given a certain thickness. When the indicator is 1 it simply means that the probability for the spacecraft to encounter the Magnetopause (or the Bow-Shock) is high. When the indicator is 0 the probability to encounter the boundary is small.

2.2 THE RADIATION BELT

The Radiation Belt is defined by its outermost boundary. This outer boundary is defined by the apex of the dipole field line (L = 6). The lower boundary is defined by a sphere of radius 1.16 Re. The Radiation Belt is divided into two belts. An Inner Belt with high energy particles confined to a L shell (L = 3.5). An Outer Belt of low energy particles extending further out to the L shell (L = 6).

The L shell is defined by the following equation:

$$r = L \sin^2 \theta \qquad \text{for} \qquad \theta = \frac{\pi}{2}, L = r$$

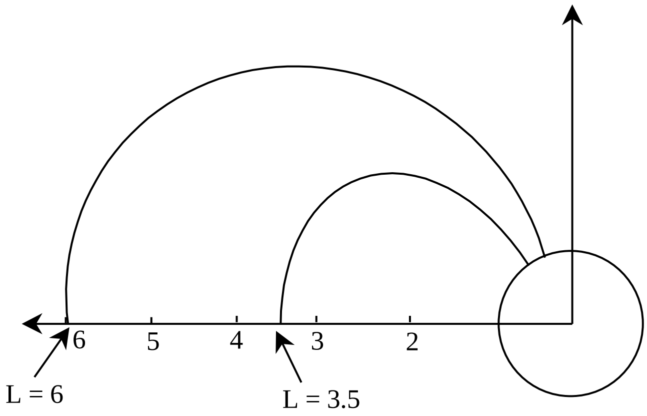

Figure 4

routine **rbelt** calculates if the spacecraft is inside the Van Allen radiation belt region.

2.3 THE PLASMASPHERE

We use the model of Chappell et al. (Ref. 2). In this model the Plasmasphere intersects the equatorial plane in approximately circular contour. For each local time the contour is defined by the apex of the dipole magnetic field line L according to Table I:

TABLE I

TGL	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h
L	4.1	4.1	4.2	4.2	4.1	4	4	4	4	4	4.15	4.3

TGL	12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h
L	4.5	4.6	4.8	5.2	6.1	6.7	6.8	6.6	6.	5.4	4.8	4.4

The routine **chapel** determines if the spacecraft is inside the Plasmasphere. Routine **chapp2** calculates the dipole field line L value of the Plasmasphere boundary for a given MLT.

2.4 THE AURORAL OVAL

We use the model of Feldstein (Ref. 5) for a moderate geomagnetic activity. The Auroral Oval is defined by its poleward and equatorward boundaries. For each local time the boundary is defined by two colatitudes according to the formulae:

$$\theta_N = A_{1N}(k) + A_{2N}(k)\cos\left(t + A_{3N}(k)\right)$$
$$\theta_S = A_{1S}(k) + A_{2S}(k)\cos\left(t + A_{3S}(k)\right)$$

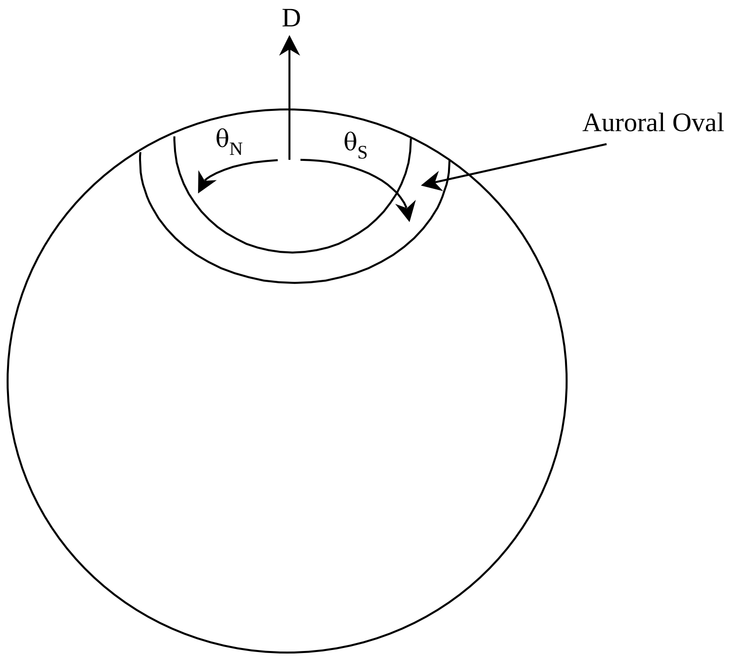

Figure 5

The routine **oval** calculates if the spacecraft is inside the Auroral Oval region.

2.5 THE POLAR CAP

The Polar Cap is defined by its boundary which is the northern boundary of the Auroral Oval. The routine **calpol** calculates if the spacecraft is inside the Polar Cap region.

2.6 THE DIFFUSE AURORAL REGION

The Diffuse Auroral region has been defined according to the work of Gussenhoven et al. (Ref. 6). For the boundary we have chosen an average geomagnetic activity level, Kp = 3. The invariant latitude of the northern boundary is given as a function of the geomagnetic local time in Table II:

TABLE II

MLT	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12
Λ_0	60.13	60.45	61.39	62.33	63.26	62.19	62.50	63.17	63.69	64.43	65.27	66.35

MLT	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	20-21	21-22	22-23	23-24
Λ_0	66.88	67.41	67.94	68.47	67.76	67.17	65.98	64.91	63.73	63.02	62.56	61.59

This table has been obtained from Table 2 of Gussenhoven et al. using the formula:

$$\Lambda = \Lambda_o + \alpha Kp$$

The northern boundary of the Diffuse Auroral region is also the southern boundary of the Auroral Oval. The routine **gussen** calculates if the spacecraft is inside this region.

2.7 THE CUSP

We have chosen the following definition:
A point belongs to the Cusp region if its conjugate on ground is located between meridians 8h MLT and 16h MLT and parallels of 75°, 80° geomagnetic latitude. The routine **cusp** calculates if the spacecraft is inside the Cusp region.

2.8 THE NEUTRAL SHEET

Fairfield (Ref. 4) has defined a simple model of a wharped Neutral Sheet. The Neutral Sheet is located in the tail beyond a circle of radius H_o sin χ, where H_o is the hinging distance and χ is the tilt angle. For a given solar magnetospheric Y coordinate the location δ_Z of the Neutral Sheet above the solar magnetospheric equatorial plane is:

$$\delta_Z = \left[(H_O + D)\left(1 - Y^2 / Y_o^2\right)^{1/2} - D \right] \sin \chi \qquad \text{for} \qquad Y < Y_o$$

and

$$\delta_Z = -D \sin \chi \qquad \text{for} \qquad Y > Y_o$$

We have H_o = 10.5 Re, D = 14 Re, Y_o = 22 R_e.

The shape and the location of the Neutral Sheet are shown in the following figures. We have chosen a thickness of 1 Re for the Neutral Sheet. The routine **posns** developed for the CLUSTER experiment Whisper calculates the distance to the Neutral Sheet. Routine **posnsh** developed for INTERBALL calculates the distance to the Neutral Sheet and indicates if the spacecraft is in the Neutral Sheet region, assuming a thickness of 1 Re for this region.

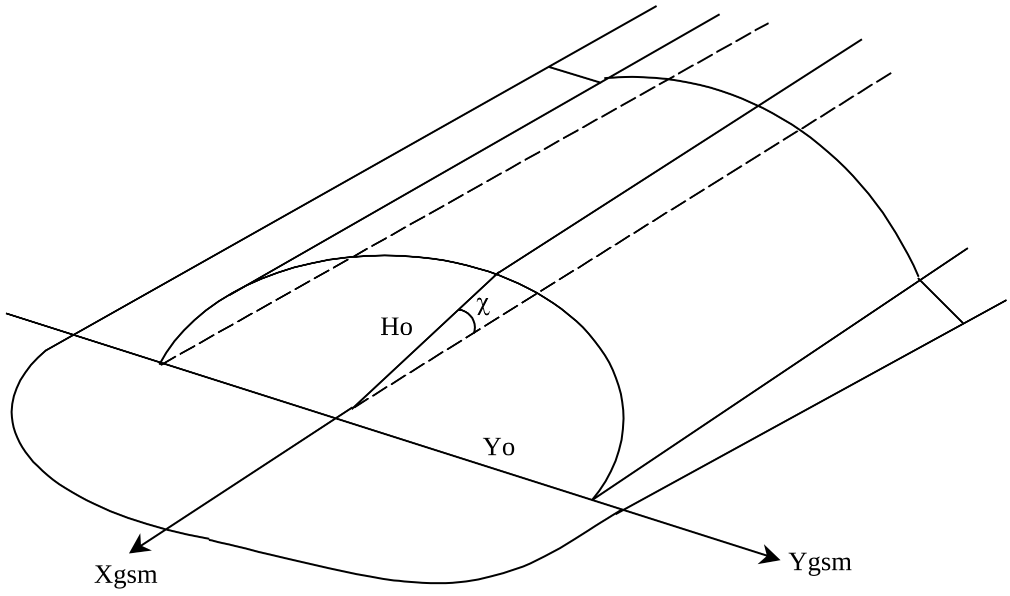

Figure 6

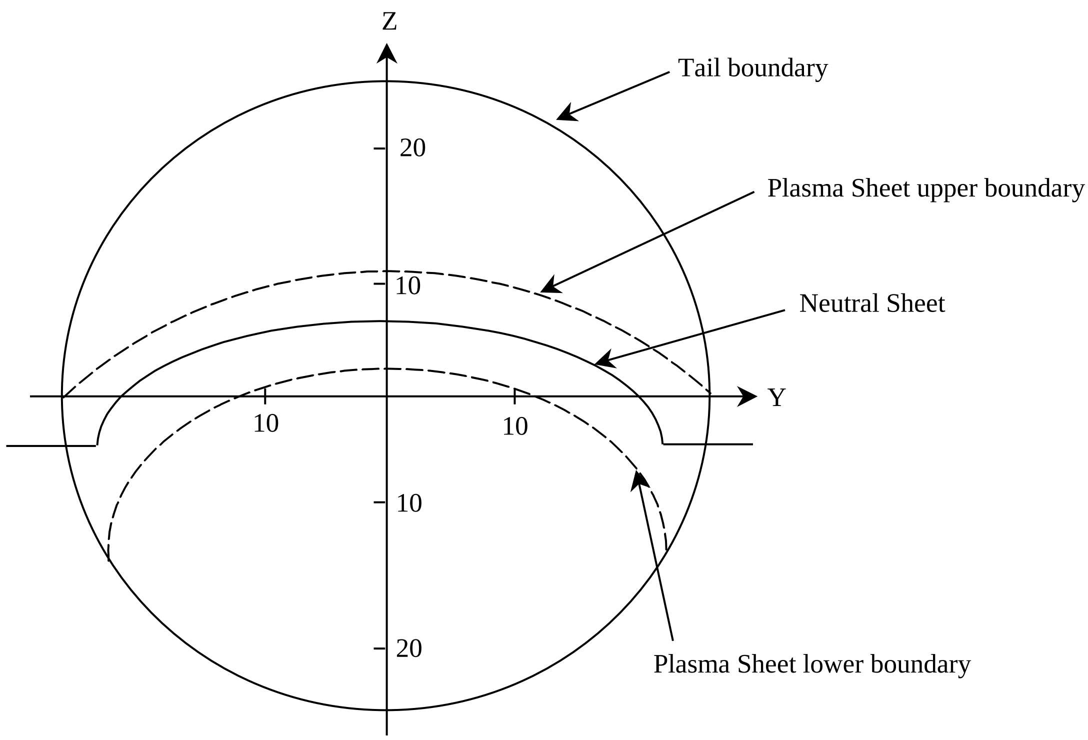

Figure 7

2.9 THE PLASMA SHEET

We define the Plasma Sheet with respect to the Neutral Sheet. If Z_{NS} is the location of a point of the Neutral Sheet the locations of the upper and lower boundaries of the Plasma Sheet are given by the following formulae:

$$Z_{PSU} = Z_{NS} + a_1 \left|Y_{GSM}\right| + b_1$$
$$Z_{PSL} = Z_{NS} - a_1 \left|Y_{GSM}\right| - b_1$$

Where Z_{PSU}, Z_{PSL} are respectively the Z coordinate of the upper and lower boundaries of the Plasma Sheet. We have chosen a_1 = 0.186 and b_1 = 3. The routine **pospsh** calculates if the spacecraft is inside the Plasma Sheet (INTERBALL).

2.10 THE MAGNETOPAUSE

We use the Sibeck model (Ref. 7). The shape of the Magnetopause is given in an axisymmetric coordinate system in the Solar Wind coordinate system:

$$R^2 + Ax^2 + Bx + C = 0$$

where *R* is the radius vector to the surface and perpendicular to the x axis. *A*, *B*, *C* are constants which depend on the Solar Wind pressure. These constants are given in Table III:

TABLE III

Pressure	A	B	C	r_b
0.54-0.87nPa	0.19	19.3	-272.4	12.6
0.87-1.47nPa	0.19	18.7	-243.9	11.7
1.47-2.60nPa	0.14	18.2	-217.2	11.0
2.60-4.90nPa	0.15	17.3	-187.4	10.0
4.90-9.90nPa	0.18	14.2	-139.2	8.8

For the probabilistic approach we have chosen two boundaries or two subsolar distances r_b = 11.7 and r_b = 10.0 for the INTERBALL project. We have the two equations:

$$R^2 + A(4)x + B(4)x + C(4) = P_1$$
$$R^2 + A(2)x + B(2)x + C(2) = P_2$$

The spacecraft is inside the Magnetosphere when $P_1 < 0$. The spacecraft crosses the Magnetopause when $P_2 < 0$ and $P_1 > 0$. The spacecraft is in the Solar Wind when $P_2 > 0$. The routine **mpsib** calculates if the spacecraft is in the Sibeck Magnetopause region (INTERBALL).

2.11 THE SHABANSKY MAGNETOPAUSE:

The Shabansky Magnetopause is an axisymmetric paraboloid (Ref. 1):

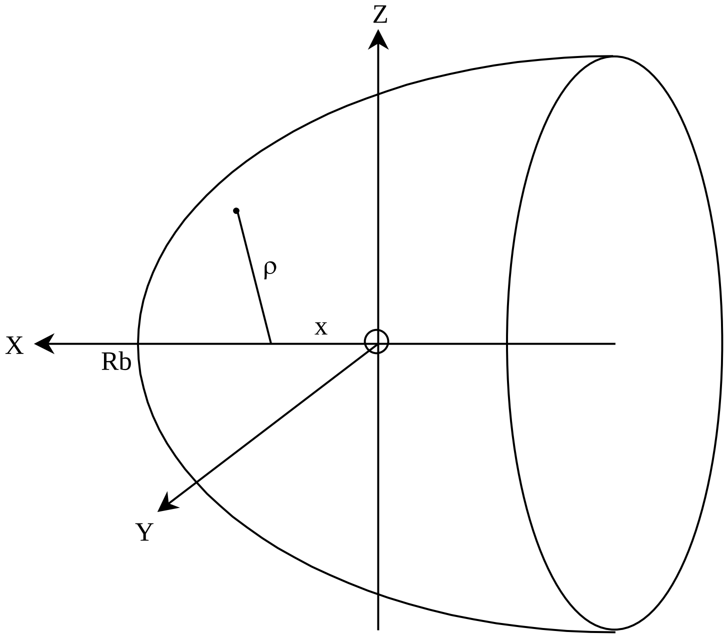

Figure 8

The surface of this paraboloid is given by the equation:

$$\frac{x}{R_b} + \frac{\rho^2}{2R_b^2} = 1$$

Where R_b is the subsolar distance and ρ is the axial distance

$$\rho^2 = y^2 + z^2$$

From the definition we get:

$$\rho = \sqrt{2R_b\left(x - R_b\right)}$$

A point (*x*, *y*, *z*) is in the Shabansky Magnetosphere if it satisfies the two conditions:

$$x < R_b \qquad \text{and} \qquad \sqrt{y^2 + z^2} < \rho(x)$$

The routine **mpause** calculates if a point if a point is inside or outside the Magnetosphere.

2.12 THE BOW-SHOCK

The Bow-Shock model is taken from Fairfield (Ref. 3). The Bow-Shock surface is defined in a Solar Wind coordinate system, i.e., a solar ecliptic system corrected from Aberration. The surface equation is:

$$y^2 + Axy + Bx^2 + Cy + Dx + E = 0$$

where x is the Solar Wind coordinate in the Sun direction and y is the radial coordinate. The Bow-Shock is axisymmetric. The coefficients given by his Table II are:

$$A = 0.0296,\ B = -0.0381,\ C = -1.280,\ D = 45.644,\ E = -652.10$$

The above equation can be simplified by a rotation U. We introduce the new coordinates X, Y in the following way:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos U & -\sin U \\ \sin U & \cos U \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$

The surface equation can be rewritten as:

$$\begin{aligned} & X^2 \left(\sin^2 U + B\cos^2 U + A\sin U \cos U\right) + Y^2 \left(\cos^2 U + B\sin^2 U - A\sin U \cos U\right) \\ & + XY \left(2\sin U \cos U - 2B\sin U \cos U - A\sin^2 U + A\cos^2 U\right) + X \left(C\sin U + D\cos U\right) \\ & + Y\left(C\cos U - D\sin U\right) + E = 0 \end{aligned}$$

We choose U in order to cancel the XY term. We get:

$$\tan 2U = \frac{A}{B-1}$$

We get U = -0°.817

After the rotation performed for the Aberration a second rotation U is thus performed as shown below:

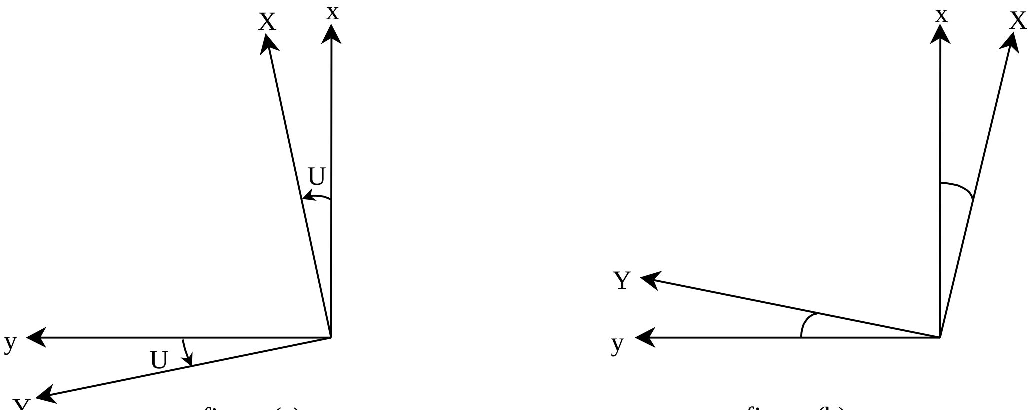

figure (a) figure (b)

U is counted positive in a rotation from x to y. U = -0.°817 corresponds to figure

Figure 9

We can write the last equation as:

$$A_r X^2 + B_r Y^2 + C_r Y + D_r X + E_r = 0$$

where

$$A_r = \sin^2 U + B\cos^2 U + A\sin U\cos U$$
$$B_r = \cos^2 U + B\sin^2 U - A\sin U\cos U$$
$$C_r = C\cos U - D\sin U$$
$$D_r = C\sin U + D\cos U$$
$$E_r = E$$

The constants A_r, B_r, C_r, D_r can be calculated. We obtain:

$$A_r = -0.03811,\quad B_r = 1.000211,\quad C_r = -0.62933,\quad D_r = 45.657607,\quad E_r = -652.1$$

The aberrated coefficients are calculated in the routine **aberrm**.

The equation above can be simplified further, dividing by B_r we obtain the equation:

$$A_n X^2 + Y_n^2 + D_n X + E_n = 0$$

where $$A_n = \frac{A_r}{B_r},\quad Y_n = Y + \frac{1}{2}\frac{C_r}{B_r},\quad D_n = \frac{D_r}{B_r},\quad E_n = \frac{E_r}{B_r} - \frac{1}{4}\frac{C_r^2}{B_r^2}$$

We make the following approximation:

$$A_n = A_r,\ \ D_n = D_r,\ \ E_n \cong E_r - \frac{1}{4} C_r^2 \cong E_r$$

For convenience in the calculations we also set $Y_n = Y_r$ which induces a systematic error of 0.3 Re in Y. The Bow-Shock general equation can be deduced as it has now axial symmetry:

$$A_n X^2 + R^2 + D_n X + E_n = 0$$

with $R^2 = Y^2 + Z^2$

In the INTERBALL we introduce a probabilistic approach in the following way. We define a cubic volume around the spacecraft location:

Xin = X - 1	Yin = Y - 1	Zin = Z - 1
Xout = X + 1	Yout= Y + 1	Zout =Z + 1

We have two equations:

$$A_n\ X_{in}^2 + Y_{in}^2 + D_n\ X_{in} + E_n = F_1$$
$$A_n\ X_{out}^2 + Y_{out}^2 + D_n\ X_{out} + E_n = F_2$$

If $F_2 < 0$ the spacecraft has not yet encountered the Bow-Shock. If $F_1 \leq 0$ and $F_2 \geq 0$ the spacecraft will probably encounter the Bow-Shock. If $F_1 > 0$ the spacecraft has probably crossed the Bow-Shock and is in the Solar Wind. These calculations are performed in **bwshff**. For CLUSTER we don't use a probabilistic approach.

2.13 SIMILARITIES BETWEEN THE EQUATIONS OF THE TWO SURFACES

From the previous paragraphs we infer that the Magnetopause and the Bow-Shock have more or less the same shape. If we adopt a common general equation we can write for the shape of any of the two surfaces:

$$\rho^2 + ax\rho + bx^2 + c\rho + dx + e = 0$$

The constant for the two surfaces are given in the following Table:

Constant	Magnetopause	Bow-Shock	Term
a	0.	0.0296	$x\rho$
b	0.14	-0.0381	x^2
c	0.	-1.280	ρ
d	18.2	45.644	x
e	-217.2	-652.10	

For the Magnetopause we have chosen the constants for moderate Solar Wind conditions. For the Bow-Shock the constants were taken in Ref 7.

2.14 INTRODUCING THE DISPLACEMENT OF THE BOW-SHOCK

As suggested by J.C. Trotignon for CLUSTER project we define the subsolar distance of the Bow-Shock as a function of the subsolar distance of the Magnetopause according to formula:

$$r_{bs} = r_b \;/\; 0.726$$

For simplicity we consider a pure parabolic surface for the Magnetopause for the determination of some constants. The equation for this Magnetopause surface is:

$$\rho = p\sqrt{x_o - x}$$

We have $\rho = 25.5$ for $x = 0$ and $\rho = 0$ for $x = 14.46$. We get easily $p = 6.706$.
From the preceding equation we can derive the constant e:

$$\rho^2 + p^2\, x - p^2\, x_0 = 0$$

The constant $e = -P^2 x_0$ thus $e = -(r_{bs}) \times 45$.

We summarize the results for various Solar Wind conditions. We give the index isw = 1 for very quiet Solar Wind and isw = 5 for very disturbed conditions. Results are shown in the Table below:

isw	1	2	3	4	5
r_b	8.8	10.0	11.0	11.7	12.6
r_{bs}	12.12	13.77	15.15	16.11	17.3
e_{bs}	545.4	619.65	681.75	724.95	778.5

The other constants have not been changed.

2.15 THE MAGNETOSHEATH

It is the region between the Bow-Shock and the Magnetopause. From the equations written previously a probabilistic approach is taken for INTERBALL and a deterministic approach is taken in CLUSTER. For INTERBALL the Magnetosheath region corresponds to $P_2 > 0$ (spacecraft out of the Magnetopause) and to $F_2 < 0$ (Bow-Shock not yet crossed by the spacecraft). The routine **msheath** for INTERBALL calculates if the spacecraft is inside the Magnetosheath region or inside the Bow-Shock region or inside the Magnetosphere. For CLUSTER another solution has been found which implies the calculation the distance of the spacecraft to the different boundaries. When the spacecraft is between the boundary and the Earth the distance is set negative. It is positive otherwise. Thus the spacecraft inside the Magnetosheath corresponds to a positive distance to the Magnetopause and a negative distance to the Bow-Shock.

2.16 THE SOLAR WIND

For INTERBALL the spacecraft is in the Solar Wind when $F_1 > 0$. For CLUSTER the distance to the Bow-Shock must be positive. The routine **bwshff** determines if the spacecraft is in the Solar Wind region (INTERBALL).

2.17 THE MAGNETOPAUSE MODEL OF SHABANSKY AND THE FIELD LINE ESCAPE

For some external magnetic field models field lines can escape in the day side of the Magnetosphere. It is necessary to stop the field line tracing in this case and a routine check is introduced at each step of the field line calculation. We have introduced the simple Magnetopause mlodel of Alekseev and Shabansky described earlier. The distance of a point of the Shabansky Magnetopause to the x axis is given by:

$$\rho_{mp} = \sqrt{2r_b\,(r_b - x)}$$

where r_b is set to 10 or 11 Re for convenience. When for a point (x, y, z) of the field line the condition $\rho \geq \rho_{mp}$ is fulfilled the calculation is stopped ($\rho = \sqrt{y^2 + z^2}$).

2.18 THE SHADE OF THE EARTH

It is sometimes useful to know if the spacecraft is in the Shade of the Earth or not. The routine **cahsl** calculates this possibility.

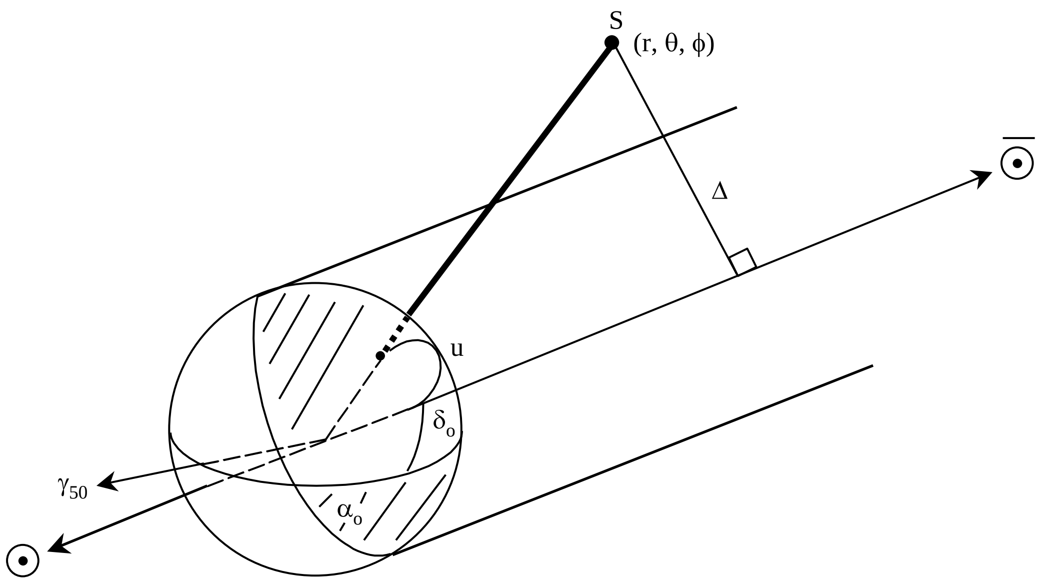

Figure 10

The position of the Sun is given by its celestial coordinates α_Θ, δ_Θ. The position of the spacecraft is given by its celestial coordinates α_s, δ_s. The position of the anti-Sun is given by the celestial coordinates: α_o, δ_o where $\alpha_o = \alpha\Theta + \pi$, $\delta_o = -\ \delta_\Theta$.

It is possible to calculate the spherical angle u between the axis of the umbra cylinder and the direction of the spacecraft:

$$\cos u = \sin\delta_O \sin\delta_S + \cos\delta_O \cos\delta_S \cos\left(\alpha_S - \alpha_O\right)$$

The distance of the spacecraft to the umbra cylinder is therefore $\Delta = r \cos u$. If $\Delta \leq R_e$ and $u \leq \pi / 2$ the spacecraft is inside the Shade of the Earth. In the other case it is outside

2.19 THE MAGNETOSPHERE AS A UNIQUE ENVIRONMENT

In this chapter we have considered all the regions encompassed by a high excentricity spacecraft such as the TAIL probe of INTERBALL project. It is tempting to assemble all these parts into one general routine which can give for any position of the spacecraft the related magnetospheric region. This routine, **posmag,** was developed for INTERBALL. This routine can associate an index 0 or 1 to any of the 15 regions of the Magnetosphere, besides other calculations (L, MLT,.....).

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Ref. 6 M.S. Gussenhoven, D.A. Hardy and N. Heinemann: Systematics of the Equatorward Diffuse Auroral Boundary, J. Geophys. Res., 88, A7, p 5692, 1983

Ref. 7 Sibeck D.G., R.E. Lopez and E.C. Roelof: Solar Wind Control of the Magnetopause Shape, Location and Motion, J. Geophys. Res., 96, 5489, 1991

3. DISTANCES TO THE BOUNDARIES

3.1 INTRODUCTION

For the CLUSTER project and the Whisper Experiment we have developed a series of routines which calculate the distances to the boundaries. These boundaries depend on the Solar Wind pressure (Bow-Shock, Magnetopause) or not (Plasmapause). Except for the distance to the Neutral Sheet we have calculated the distance to the boundary as the minimum distance between a spacecraft location and the surface.

3.2 DISTANCES TO THE MAGNETOPAUSE AND THE BOW-SHOCK

These two boundaries are paraboloids. The equation of the surface is defined as:

$$A_n x^2 + \rho^2 + D_n x + E_n = 0 \tag{1}$$

Where A_n, D_n, E_n are obtained in the Solar Wind coordinate system. Taking into account the axial symmetry, the distance between the spacecraft and a point on the surface is:

$$d^2 = (\rho - \rho_1)^2 + (x - x_1)^2 \tag{2}$$

where (x_1, ρ_1) are the coordinates of the spacecraft. We define the distance as the minimum of d. taking the derivative with respect to x we get:

$$d' = \frac{(\rho - \rho_1)\rho' + (x - x_1)}{\sqrt{(\rho - \rho_1)^2 + (x - x_1)^2}} = 0 \tag{3}$$

The derivative ρ' can be obtained from equation (1):

$$\rho' = -\frac{D_n + 2A_n x}{2\rho} \tag{4}$$

Inserting this result in the previous equation we get:

$$2\rho(x - x_1) - (\rho - \rho_1)(D_n r + 2A_n x) = 0 \tag{5}$$

ρ can be extracted from (1) and equation (5) can be rewritten as:

$$A_n x^2 + \frac{\rho_1^2 (D_n + 2A_n x)}{[2(x - x_1) - (D_n + 2A_n x)]^2} + D_n x + E_n = 0 \tag{6}$$

This equation can be solved numerically. The distances are counted positive if the surface is between the spacecraft and the Earth and negative if the spacecraft is located between the surface and the Earth. The distance to the Magnetopause or the Bow-Shock is calculated in the routine **caldis** for the CLUSTER project, and the intensity of the Solar Wind is taken into account. It is also possible to calculate the distance to the Shabansky Magnetopause parabola using the cardan algorithm to obtain the roots of a third degree equation. The calculations are performed in routine **ddparab**.

3.3 DISTANCE TO THE NEUTRAL SHEET

The Neutral Sheet position has been defined using the formulae of Fairfield (Ch 2, Ref 4). To locate the spacecraft with respect to the Neutral Sheet we define the distance of the spacecraft to the Neutral Sheet as:

$$d = zgsm - z_{nsh} \qquad (7)$$

where *zgsm* is calculated in the GSM coordinate system and z_{nsh} is the *z* coordinate of the Neutral Sheet at point *xgsm*, *ygsm*:

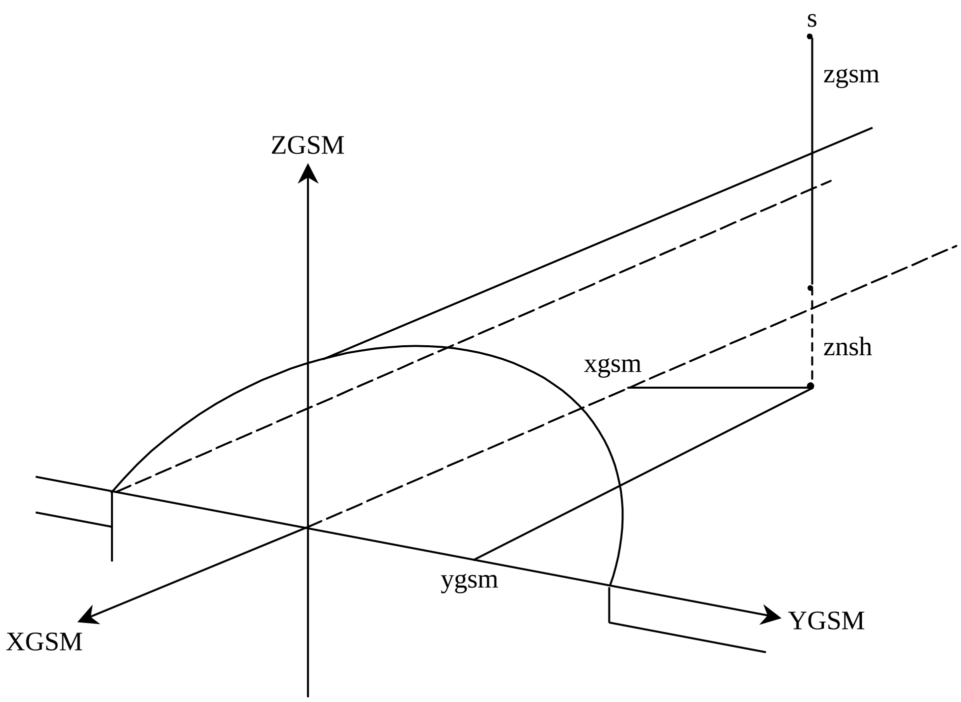

Figure 1

The distance is counted positive when the spacecraft is above the Neutral Sheet and the distance is counted negative below. The calculation is done in routine **posns** (CLUSTER) and in routine **posnsh** (INTERBALL).

3.4 DISTANCE TO THE PLASMA SHEET

The distance to the Plasma Sheet for CLUSTER uses the model developed for INTERBALL where we were interested only in verifying if the spacecraft was in the region:

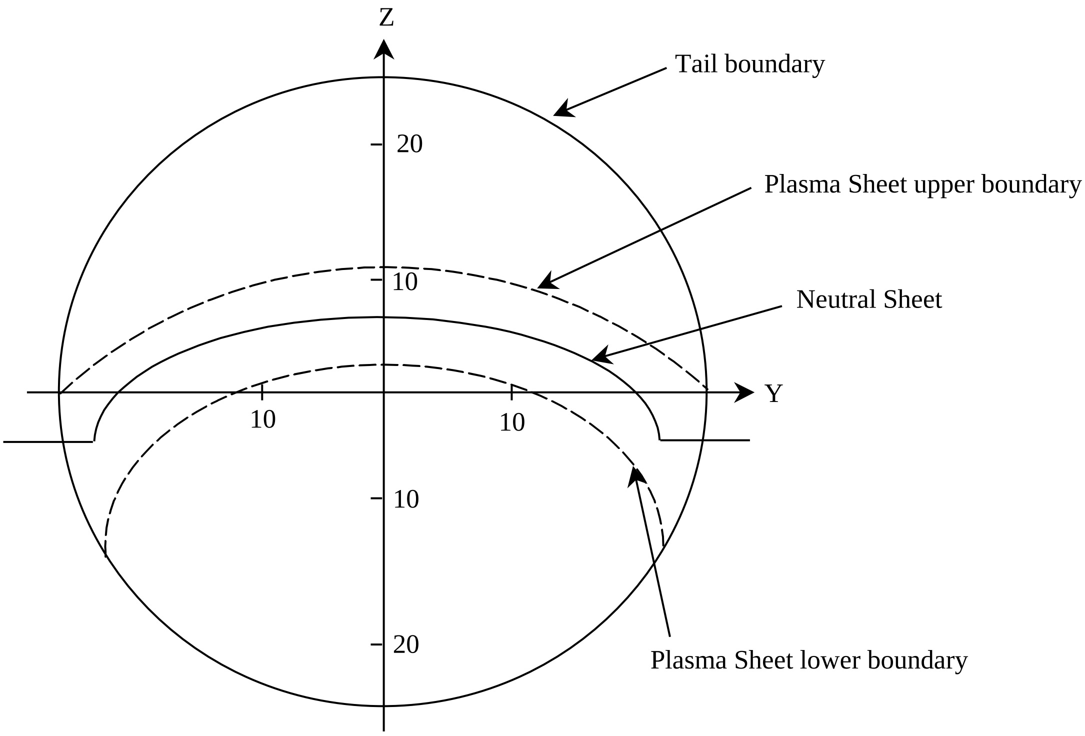

Figure 2

The upper and lower boundaries of the Plasma Sheet were defined by the following equations:

$$Z_{PSU} = Z_{NS} + a_1 \left|Y_{GSM}\right| + b_1$$
$$Z_{PSL} = Z_{NS} - a_1 \left|Y_{GSM}\right| - b_1$$

For CLUSTER the distance to the northern boundary of the Plasma Sheet is defined as:

$$dzpshn = Z_S - Z_{PSU}$$

The distance to the southern boundary of the Plasma Sheet is defined as:

$$dzpshs = Z_S - Z_{PSL}$$

These calculations are performed in routine **posps**.

3.5 DISTANCE TO THE PLASMAPAUSE

The Plasmasphere boundary has been defined by Chappell (Ref. 1) as a distorted shell where the field line parameter L is local time dependent. The shape of the Plasmapause is a continuum of dipole field lines of various L. For a given spacecraft location *xgsm*, *ygsm*, *zgsm* and a given Epoch (U.T.) it is possible to calculate the magnetic local time of the spacecraft. From Table II in the previous chapter it is possible to calculate the L parameter of the corresponding field line by interpolation between two nearby local times. Once L is obtained the distance *d* can be calculated.

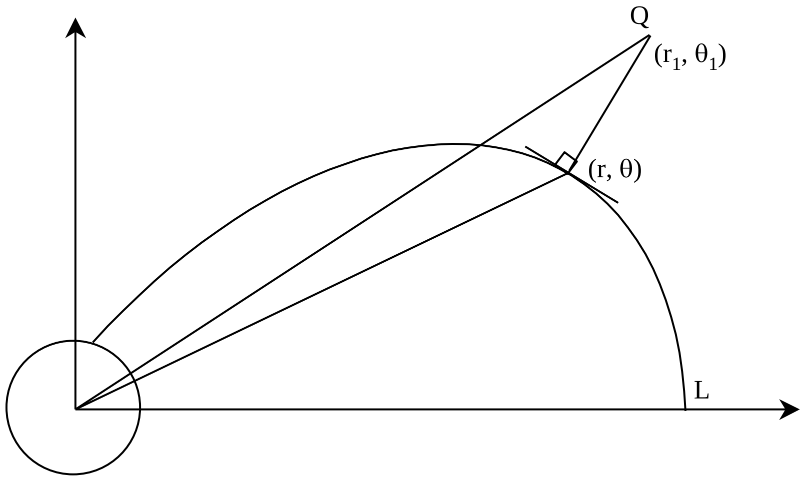

Figure 3

The distance *d* between the spacecraft located in (r_1, θ_1) and the Plasmapause is obtained for the point (r, θ) which corresponds to *d* minimum:

$$d^2 = r^2 + r_1^2 - 2rr_1 \cos(\theta - \theta_1) \tag{8}$$

We have for a dipole field line: $r = L \sin^2 \theta$

Taking the derivative with respect to θ we obtain:

$$d'_\theta = L \sin^2 \theta \left[4L\sin^2 \theta \cos\theta - 4r_1 \cos\theta \cos(\theta - \theta_1) - 2r_1 \sin\theta \sin(\theta - \theta_1)\right] \tag{9}$$

$d'_\theta = 0$ will correspond to *d* minimum. Equation (9) can be solved numerically. The calculation is performed in routine **dchapp** (CLUSTER).

3.6 DISTANCES, A GENERAL ALGORITHM

For the WHISPER experiment onboard CLUSTER we have to calculate the distances to the different boundaries, Magnetopause, Bow-Shock, Plasmasphere, Neutral Sheet, northern and southern Plasma Sheet boundaries. The calculation to this complete set of boundaries is performed in routine **clusdis** which assembles the different routines developed for CLUSTER and described above.

REFERENCES

Ref. 1 C.R. Chappell, K.K. Harris and G.W. Sharp: The Dayside of the Plasmasphere, J.Geophys.Res.,76,31, p 7632

4. INTERNAL MAGNETIC FIELD MODELS

4.1 INTRODUCTION

Internal magnetic field models are derived from measurements performed in ground stations, ships and aboard low-Earth orbit spacecraft. Ground measurements have two advantages:

- They are and have been made during a long period.
- They are performed in good conditions (magnetic observatories).

Their major disadvantage is the presence of magnetic perturbations generated by localized crust anomalies which can lead to errors as high as 100 to 400 nanoteslas (nT). Magnetic observatories are not available on oceans and seas and the magnetic measurements are performed by ships.
Spacecraft measurements offer a good coverage of the whole Earth in a rather short time. The on-board magnetometers have presently a high sensitivity (better than 0.1 nT). The final precision depends mainly on the local perturbations due to the spacecraft itself or the uncertainty in the attitude determination.
Present magnetic field models achieve a precision of 20 nT.

4.2 THE SPHERICAL HARMONICS EXPANSION

Present models of the internal magnetic field usually neglect the contributions (magnetic field effects) of the ionospheric currents and of the ring current. The magnetic field is therefore derived from a scalar potential and is also divergence free:

$$\vec{B} = -\nabla V \qquad \text{and} \qquad \nabla \cdot B = 0 \tag{1}$$

which leads to $\nabla^2 V = 0$

In spherical coordinates the Laplace equation can be written:

$$\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2 V}{\partial\varphi^2}\right]=0 \tag{2}$$

We separate the variables:

$$V = f(r)Y(\theta,\varphi) \tag{3}$$

We obtain two different equations:

The Euler equation: $$\frac{\partial}{\partial r}\left(r^2 f(r)\right)-\lambda f(r)=0 \tag{4}$$

and the spherical harmonics equation: $$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial Y}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2 Y}{\partial y^2}+\lambda Y=0 \qquad (5)$$

The solution of the Euler equation for the internal magnetic field can be written:

$$f(r)=\frac{B}{r^{\alpha+1}} \qquad (6)$$

The spherical harmonics equation can be split further:

$$Y(\theta,\varphi)=g(\mu)h(\varphi) \qquad (7)$$

where $\mu = \cos\theta$

We get:

$$h(\varphi)\frac{\partial}{\partial\mu}\left[\left(1-\mu^2\right)g'(\mu)\right]+\frac{1}{1-\mu^2}g(\mu)h''(\varphi)+\alpha(\alpha+1)g(\mu)h(\varphi)=0 \qquad (8)$$

We consider harmonic functions with period 2π. In this case $\frac{h''}{h}=-n^2$ (modulo some constant). The solution h (φ) has the following form:

$$h(\varphi)=M\cos m\varphi+N\sin m\varphi$$

Function $g(\mu)$ satisfies the following differential equation:

$$\left(1-\mu^2\right)g''-2g\mu'+\left[\alpha(\alpha+1)+\frac{n^2}{1-\mu}\right]g_2=0 \qquad (9)$$

In this particular case when n = 0, α is an integer, $g(\mu)$ satisfies the equation:

$$\left(1-\mu^2\right)g''-2g\mu'+\alpha(\alpha+1)g=0 \qquad (10)$$

The solution of this equation is the Legendre polynomial $P_k(\mu)$. Thus for n = 0 the solution has the form:

$$V_k=\frac{B}{r^{k+1}}P_k(\cos\theta) \qquad (11)$$

when the longitude φ is absent the potential V can be expanded in zonal harmonics. The Legendre functions can be expressed as:

$$P_0 = 1\,,\; P_1 = \cos\theta\,,\; P_2 = \frac{1}{4}\left(3\cos^2\theta + 1\right),\; P_3 = \frac{1}{8}\left(5\cos^3\theta + 3\cos\theta\right),... \qquad (12)$$

In a more general way:

$$P_n(\theta) = \frac{1.3.5......(2n-1)}{2.4.6.....(2n)}\left\{2\cos n\theta + 2\frac{1.n}{1.(2n-1)}\cos(n-2)\theta + 2.\frac{1.3.n(n-1)}{1.2.(2n-1)(2n-3)}\cos(n-4)\theta + ...\right\}$$

We have:

$$\int_{-1}^{+1} P_n(\mu)\,P_m(\mu)\,d\mu = \begin{cases} 0 \; if \; n \neq m \\ \dfrac{2}{2n+1} if \; n = m \end{cases} \qquad (13)$$

Functions $\left[\frac{2n+1}{2}\right]^{1/2} P_n(\mu)$ are normalized.

Any sum of functions $\frac{B_k}{r^{k+1}} P_k(\cos\theta)$ is also a solution of equation $\nabla^2 V = 0$ when n is zero.

The most general solution in the axisymmetric case is thus:

$$V = \sum_0^{\infty} \frac{B_k}{r^{k+1}} P_k(\cos\theta) \qquad (14)$$

when n is different from zero the equation (9) is an associated Legendre equation. The solution of this equation is an associated Legendre function $P_{n,m}(\mu)$ defined as:

$$P_{n,m}(\mu) = \left(1-\mu^2\right)^{m/2} \frac{d^m}{d\mu^m} P_n(\mu) \qquad (15)$$

The general solution of $\nabla^2 V = 0$ has the following form:

$$V = \sum_0^{\infty} \left(\frac{1}{r}\right)^{n+1} \sum_{m=0}^{n} \left(M \cos m\varphi + N \sin m\varphi\right) P_{n,m}(\cos\theta) \qquad (17)$$

The associated Legendre functions can be written as:

$$P_{n,m}(\cos\theta) = \sin^m\theta \frac{d^m}{d(\cos\theta)^m} P_n(\cos\theta) \qquad (18)$$

We get:

$$P_{1,1} = \sin\theta\,,\ P_{2,1} = \frac{3}{2}\sin 2\theta\,,\ P_{2} = 3\sin^2\theta \tag{19}$$

more generally:

$$P_{n,m}(\theta) = \frac{(2n)!}{2^n.n!(n-m)!}\sin^m\theta\left[\cos^{n-m}\theta - \frac{(n-m)(n-m-1)}{2(2n-1)}\cos^{n-m-2}\theta + \cdots\cdots\cdots\right] \tag{20}$$

The $P_{n,m(\cos\theta)}$ can be normalized.

$$\int_{-1}^{+1} P_{n,m}(\mu)\,P_{n,m}(\mu) = \begin{cases} 0 \ \ for\ n \neq n \\ \dfrac{2}{2n+1}\dfrac{(n+m)!}{(n-m)!}\ \ for\ n = n \end{cases} \tag{21}$$

A truly normalized associated function should have the normalizing factor:

$$\sqrt{\frac{2n+1}{2}\frac{(n-m)!}{(n+m)!}} \qquad \text{for } m \neq 0 \tag{22}$$

and $$\sqrt{\frac{2n+1}{2}} \qquad \text{for } m = 0 \tag{23}$$

In fact Schmidt defined new normalizing constants. The associated Legendre functions $P_{n,m}$ (cos θ) cos $m\varphi$ and $P_{n,m}$ (cos θ) sin $m\varphi$ are orthogonal on a sphere. Surface integration on a sphere of radius a gives:

$$\frac{1}{4\pi}\int_{\theta=0}^{\theta=\pi}\int_{\varphi=0}^{\varphi=\pi} P_{n,m}(\theta)\begin{Bmatrix}\cos\\ \sin\end{Bmatrix} m\varphi P_{n',m'}(\theta)\begin{Bmatrix}\cos\\ \sin\end{Bmatrix} m'\varphi \sin\theta d\theta d\varphi = 0 \tag{24}$$

When $n = n'$ and $m = m' = 0$ we get (13).
When $n = n'$ and $m = m' \neq 0$ we get:

$$\frac{1}{4\pi}\int_{\theta=0}^{\theta=\pi}\int_{0}^{2\pi}\left[P_{n,m}(\theta)\begin{Bmatrix}\cos\\ \sin\end{Bmatrix} m\varphi\right]^2 \sin\theta d\theta d\varphi = \frac{1}{2(2n+1)}\frac{(n+m)!}{(n-m)!}$$

Schmidt used this fact to define the following constants:

$$P_n^m(\theta) = P_{n,m}(\theta) \qquad \text{when } m = 0 \tag{25}$$

$$P_n^m(\theta) = \sqrt{2\frac{(n-m)!}{(n+m)!}}\,P_{n,m}(\theta) \qquad \text{when } m \neq 0 \tag{26}$$

In this case:

$$\frac{1}{4\pi}\int_{\theta=0}^{\theta=\pi}\int_{0}^{2\pi}\left[P_n^m(\theta)\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}m\varphi\right]^2 \sin\theta d\theta d\varphi = \frac{1}{(2n+1)} \tag{27}$$

These functions are not really normalized. Their main advantage lies in the fact that the constants A_n^m, B_n^m give roughly the size of the different harmonics.

The associated Legendre polynomials have the following properties:

- $P_n^m(\theta)$ has $(n - m)$ real roots between $\theta = 0$ and $\theta = \pi$
- if $(n - m)$ is even or odd $P_n^m(\theta)$ is symmetric or antisymmetric with respect to the equator.
- the $P_n^m(\theta)$ can be written:

$$P_n^m(\theta) = p_n^m \sin^m\theta(\cos\theta - \cos\theta_1)(\cos\theta - \cos\theta_2)\cdots\cdots(\cos\theta - \cos\theta_{n-m}) \tag{28}$$

where θ_1, θ_2, ……., θ_{n-m} are the $(n - m)$ roots. The harmonics $P_n^m(\theta)\begin{Bmatrix}\cos m\varphi\\ \sin m\varphi\end{Bmatrix}$ are zero along the $(n - m)$ small latitude circles and along the $(2m)$ meridians.
For $n = 0$ we have zonal harmonics.
For $m = n$ we have sectoial harmonics.
For $n > m > 0$ we have tesseral harmonics.
The relationship between the Schmidt functions $P_{n,m}(\theta)$ and the Laplace-Gauss functions $P^{n,m}(\theta)$ is:

$$P_{n,m}(\theta) = \frac{(2n-1)!!}{(n-m)!}P^{n,m}(\theta) \tag{29}$$

where $(2n - 1)!! = 1.3.5\ldots\ldots(2n - 1)$

4.3 GENERAL EXPRESSION OF THE MAGNETIC FIELD IN SPHERICAL HARMONICS

The potential of the magnetic field can be written as:

$$V = a\sum_{1}^{n\max}\left(\frac{a}{r}\right)^{n+1}\sum_{0}^{n}\left(g_n^m\cos m\varphi + h_n^m\sin m\varphi\right)P_n^m(\theta) \tag{30}$$

where r, θ, φ are the geocentric coordinates. r is counted in kilometers, a is the geomagnetic earth radius (6371.2 km), θ is the colatitude and φ is the longitude. The components of the geomagnetic field are:

$$B_r = -\frac{\partial V}{\partial r} = \sum_{1}^{n\max} \left(\frac{a}{r}\right)^{n+2} (n+1) \sum \left(g_n^m \cos m\varphi + h_n^m \sin m\varphi\right) P_n^m(\theta) \tag{31a}$$

$$B_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = -\sum_{1}^{n\max} \left(\frac{a}{r}\right)^{n+2} \sum_{0}^{n} \left(g_n^m \cos m\varphi + h_n^m \sin m\varphi\right) \frac{\partial P_n^m(\theta)}{\partial \theta} \tag{31b}$$

$$B_\varphi = -\frac{1}{r\sin\theta}\frac{\partial V}{\partial \theta} = \frac{1}{\sin\theta} \sum_{1}^{n\max} \left(\frac{a}{r}\right)^{n+2} \sum m \left(g_n^m \sin m\varphi - h_n^m \cos m\varphi\right) P_n^m(\theta) \tag{31c}$$

The g_n^m and h_n^m are the Schmidt normalized coefficients of the Earth's magnetic field. The P_n^m are the associated Legendre functions. EOS as well as Scientific Journals publish each five years the Gauss coefficients for the IGRF and DGRF.

The IGRF (Interim Geomagnetic Reference Field) is predictive while the DGRF (Definitive Geomagnetic Reference Field) corresponds to stable coefficients. For example IAGA 2015 defined the IGRF 15 and the DGRF 10.

4.4 THE LAPLACE GAUSS RECURRENT FORMULAE

The Laplace Gauss formulae are used in the software for the calculations of the internal magnetic field. To derive them we use the equation (51aa, Ref. 1, p.623):

$$(2n+1)\cos\theta P_{n,m} = (n-m+1)\, P_{n+1,m} + (n+m)\, P_{n-1,m} \tag{32}$$

which can be written:

$$(n-m+1)\, P_{n+1,m} = (2n+1)\cos\theta P_{n,m} - (n+m)\, P_{n-1,m} \tag{33}$$

If we set $n \to n - 1$ we get:

$$(n-m)P_{n,m} = (2n-1)\cos\theta P_{n-1,m} - (n+m-1)\, P_{n-2,m} \tag{34}$$

Thus:

$$P_{n,m} = \frac{(2n-1)}{n-m}\cos\theta P_{n-1,m} - \frac{n+m-1}{n-m}\, P_{n-2,m} \tag{35}$$

We introduce the relation between $P_{n,m}$ and $P^{n,m}$:

$$P_{n,m} = \frac{(2n-1)!}{(n-m)!}\, P^{n,m}\,,\ P_{n-1,m} = \frac{(2n-3)!}{(n-m-1)!}\, P^{n-1,m}\,,\ P_{n-2,m} = \frac{(2n-5)!}{(n-m-2)!}\, P^{n-2,m} \tag{36}$$

Introducing (36) into (35) we obtain:

$$P^{n,m}=\frac{(2n-3)!\,(2n-1)(n-m)!}{(n-m-1)!\,(2n-1)!\,(n-m)}\cos\theta P^{n-1,m}-\frac{(2n-5)!\,(n+m-1)(n-m)!}{(n-m-2)!\,(n-m)(2n-1)!}P^{n-2,m} \qquad (37)$$

But we can write $\qquad (2n-1)!=(2n-3)!\,(2n-1)=(2n-5)!\,(2n-3)(2n-1) \qquad (38)$

and $\qquad (n-m)!=(n-m-1)!\,(n-m)=(n-m-2)!\,(n-m-1)(n-m) \qquad (39)$

Introducing (37) and (38) into equation (37) gives:

$$P^{n,m}=\cos\theta P^{n-1,m}-K^{n,m}\;P^{n-2,m} \qquad (40)$$

where $\qquad K^{n,m}=\dfrac{(n-1)^2-m^2}{(2n-1)(2n-3)} \qquad (41)$

This formula is valid for $n > m$. For $n = m$ we start with the general formula (42):

$$P_{n,m}(\theta)=\frac{(2n)!}{2^n\,n!\,(n-m)!}\sin^m\theta\left\{\cos^{n-m}\theta-\frac{(n-m)(n-m-1)}{2(2n-1)}\cos^{n-m-2}\theta+\cdots\cdots\cdots\right\} \qquad (42)$$

For $n = m$ we get:

$$P_{n,n}(\theta)=\frac{2n!}{2^n n!}\sin^n\theta \qquad (43)$$

For n - 1 we obtain from (43): $\qquad P_{n-1,n-1}(\theta)=\dfrac{(2n-2)!}{2^{n-1}(n-1)!}\sin^{n-1}\theta \qquad (44)$

Comparing (43) and (44) we derive the following relationship (argument θ is omitted for the Legendre polynomials):

$$P_{n,n}=(2n-1)\sin\theta\,P_{n-1,n-1} \qquad (45)$$

To obtain the Laplace-Gauss relationship we take into account the formula:

$$P_{n,n}=(2n-1)!\,P^{n,n} \qquad (46)$$

Applying (46) to both members of equation (45) gives:

$$P^{n,n}=\sin\theta\,P^{n-1,n-1} \qquad (47)$$

Taking into account the general formula (43) and calculating the derivatives of equations (40) and (47) we have the complete set of equations:

$$P^{0,0} = 1 \qquad \frac{\partial P^{0,0}}{\partial \theta} = 0 \tag{48a}$$

$$P^{n,n} = \sin\theta \, P^{n-1,n-1}, \quad \frac{\partial P^{n,n}}{\partial \theta} = \sin\theta \, \frac{\partial P^{n-1,n-1}}{\partial \theta} + \cos\theta \, P^{n-1,n-1} \tag{48b}$$

$$P^{n,m} = \cos\theta \, P^{n-1,m} - K^{n,m} \; P^{n-2,m} \tag{48c}$$

$$\frac{\partial P^{n,m}}{\partial \theta} = \cos\theta \, \frac{\partial P^{n-1,n-1}}{\partial \theta} - \sin\theta \, P^{n-1,m} - K^{n,m} \; \frac{\partial P^{n-2,m}}{\partial \theta} \tag{48d}$$

with

$$K^{n,m} = \frac{(n-1)^{é} - m^2}{(2n-1)(2n-3)} \tag{48e}$$

The Schmidt normalizing functions must be converted in the gaussian system. To obtain the recurrence formulae one notices that:

from equation (29) :

$$P_{n,m} = P^{n,m} \; \frac{(2n-1)!!}{(n-m)!} \tag{49}$$

and from equation (26):

$$P_n^m = \sqrt{\frac{2(n-m)!}{(n+m)!}} \; P_{n,m} \tag{50}$$

We have:

$$P_n^m = \sqrt{\frac{2(n-m)!}{(n+m)!}} \frac{(2n-1)!!}{(n-m)!} \; P^{n,m} = S^{n,m} \; P^{n,m} \tag{51}$$

and:

$$P_n^{m-1} = \sqrt{\frac{2(n-m+1)!}{(n+m-1)!}} \; \frac{(2n-1)!!}{(n-m+1)!} \; P^{n,m-1} = S^{n,m-1} \; P^{n,n-1} \tag{52}$$

The coefficient $S^{n,m}$ of equation (51) can be rewritten as:

$$S^{n,m} = \sqrt{\frac{2(n-m+1)!}{(n+m-1)!\,(n+m)(n-m+1)}} \; \frac{(2n-1)!!\,(n-m+1)}{(n-m+1)!} \tag{53}$$

We can replace part of the above expansion by coefficient $S^{n,m-1}$ and obtain finally:

$$S^{n,m} = S^{n,m-1} \sqrt{\frac{n-m+1}{n+m}} \tag{54}$$

for $m \neq 1$. When $m = 0$ we have from equation (49):

$$P_{n,0} = P^{n,0} \frac{(2n-1)!!}{n!} = S^{n,0} P^{n,0} \tag{55a}$$

For $m = 1$ we have

$$P_n^1 = \sqrt{\frac{2(n-1)!}{(n+1)!}} P_{n,1} = \sqrt{\frac{2(n+1)!}{(n+1)!}} \frac{(2n-1)!!}{(n-1)!} P^{n,1} \tag{55b}$$

which can also be rewritten as:

$$P_n^1 = \sqrt{\frac{2(n-1)!}{(n+1)!}} \frac{n}{n!} (2n-1)!! P^{n,1} = S^{n,1} P^{n,1} \tag{55c}$$

We can find a recurrence relationship between $S^{n,1}$ and $S^{n,0}$ using equations (55a) and (55c):

$$S^{n,1} = \sqrt{\frac{2(n-1)!}{(n-1)! n(n+1)}} n S^{n,0} = \sqrt{\frac{2n}{n+1}} S^{n,0} \tag{56}$$

If we consider the more general formula for $m > 1$ we would have:

for $m = 1$

$$S^{n,1} = S^{n,0} \sqrt{\frac{n-m+1}{n+m} \times 2} \tag{57}$$

In order to bobtain a unique formula we write:

$$S^{n,m} = S^{n,m-1} \sqrt{\frac{J(n-m+1)}{n+m}} \tag{58}$$

where $J = 2$ for $m = 1$ and $J = 1$ for $m > 1$. For $m = 0$ we can deduce a relationship from equation (56):

$$S^{n,à} = S^{n-1,0} \frac{(2n-1)}{n} \tag{59}$$

4.5 THE RECURRENT FORMULAE IN THE SOFTWARE

The calculation of these formulae with the computer imply the use of arrays without null indexes. To avoid this problem it is necessary to make changes in the superscript: k = n + 1 and l = m + 1. We obtain a new set of formulae which appear in the code if we replace n,m in formulae (58) and (59):

$$S^{k,l} = S^{k,l-1} \sqrt{\frac{(k-l+1)j}{k+l-2}} \tag{60a}$$

$$S^{k,0} = S^{k-1,0} \left[\frac{2k-3}{k-1}\right] \tag{60b}$$

For the Legendre polynomials we have the following expressions:

$$P^{1,1} = 1, \qquad P^{k,k} = \sin\theta\, P^{k-1,k-1} \tag{61a}$$

$$\frac{\partial P^{1,1}}{\partial\theta} = 0, \qquad \frac{\partial P^{k,k}}{\partial\theta} = \sin\theta \frac{\partial P^{k-1,k-1}}{\partial\theta} + \cos\theta\, P^{k-1,k-1} \tag{61b}$$

$$\frac{\partial P^{k,l}}{\partial\theta} = \cos\theta \frac{\partial P^{k-1,l}}{\partial\theta} - \sin\theta\, P^{k-1,l} - K^{k,l} \frac{\partial P^{k-2,l}}{\partial\theta} \qquad \text{where} \qquad K^{k,l} = \frac{(k-2)^2 - (l-1)^é}{(2k-3)(2k-5)} \tag{61c}$$

The coefficients in the harmonic expansion have non zero superscripts, i.e., g_1^0 is labeled as $g(2,1)$ in the software. In the american software memory was saved and coefficients g_n^m and h_n^m were packed in a square matrix as follows:

$$\begin{array}{ll}
g(1,1)\, h(2,2)\, h(3,2) & h(n,2) \\
g(2,1)\, g(2,2)\, h(3,3) & h(n,3) \\
g(3,1)\, g(3,2)\, g(3,3) & h(n,4) \\
 & \\
g(n,1)\, g(n,2)\, g(n,3) & g(n,n)
\end{array} \tag{62}$$

These coefficients are stored in the common LG, column by column. The coefficient $g(1,1)$ contains a numerical factor 10 or 100 which normalizes the coefficients and these coefficients as stored as integers. The only problem with this way of arranging the (14,14) matrix is that it is a rather cumbersome exercise when one remembers that the coefficients given in the literature have different indices: For IGRF 2015 we have in IAGA the following table I:

TABLE I

g/h	n	m	IGRF 2010 coefficients in nanoteslas	secular variation in nanoteslas/yr
g	1	0	-29442,0	10,3
g	1	1	-1501,0	18,1
h	1	1	4797,1	-26,6
g	2	0	-2445,1	-8,7
g	2	1	3012,9	-3,3
h	2	1	-2845,6	-27,4
g	2	2	1676,7	2,1
h	2	2	-641,9	-14,1

In the code we have the following table II or matrix lg:

TABLE II

	l = 1	l = 2	l = 3	l = 4
k = 1	lg(1,1) = 10	lg(1,2) = 47971	lg(1,3) = -28456	lg(1,4) = -1153
k = 2	lg(2,1) = -294420	lg(2,2) = -15010	lg(2,3) = -6419	lg(2,3) = 2449
k = 3	lg(3,1) = -24451	lg(3,2) = 30129	lg(3,3) = 16767	lg(3,3) = -5384

On the lower left of the last table one recognizes the $g(k,l)$ coefficients and on the upper right one recognizes the $h(k,l)$. These coefficients are read as integers in the matrix lg(k,l) and the transformation factor from integers to real is contained in the coefficient lg(1,1).
A similar matrix must be constructed for the secular variation of the $g(k,l)$ and $h(k,l)$ i.e., the $\dot{g}(k,l)$ and $\dot{h}(k,l)$. The magnetic field is calculated with this algorithm by the routines **dgrf45_70** for epochs between 1945-1970, **dgrf70_95** for epochs between 1970-1995, **dgrf95_15** for epochs between 1995-2015, **igrf15** for epochs greater than 2015, **dgrf10** for epochs greater than 2010, **dgrf05** for epochs greater than 2005, **dgrf00** for epochs greater than 2000, **dgrf95** for epochs greater than 1995, and **gsfc65** for epochs around 1965 (Mc Ilwain L calculation).

4.6 A NEW COMPUTER CODE FOR THE CALCULATION OF THE GEOMAGNETIC FIELD

In the previous paragraph we have mentioned that the creation of the LG matrix was a difficult task and a source of errors because of the difference in the ordering of the coefficients. We have modified the computer code in order to keep the natural ordering of the coefficients published each five years by the IAGA and we have two commons, one *LG* for the coefficients in their natural order:

$$LG/\,g(1,0),g(1,1),h(1,1),g(2,0),g(2,1),h(2,1),....$$

and the common *LGT* which contains the secular derivatives. The coefficients in *LG* and in *LGT* are transformed in coefficients *gg*, *ggt* and *hh*, *hht* in the following double loop:

$$
\begin{aligned}
&\quad ntot = 0 \\
&\quad do\ 30\ k = 2, k\max \\
&\quad gg(k,l) = dble(\lg(ntot+1)) \\
&\quad ggt(k,l) = dble(\lg t(ntot+1)) \\
&\quad nm = 2*k-1 \\
&\quad do\ 20\ l = 2, k \\
&\quad inc = 2*l-2 \\
&\quad gg(k,l) = dble(\lg(ntot+inc)) \\
&\quad ggt(k,l) = dble(\lg t(ntot+inc)) \\
&\quad hh(k,l) = dble(\lg(ntot+inc+1)) \\
&\quad hht(k,l) = dble(\lg t(ntot+inc+1)) \\
&20\ continue \\
&\quad ntot = ntot+nm \\
&30\ continue
\end{aligned}
$$

with this new software the DGRF95 coefficients are stored in arrays *LG* and *LGT* in the following way:

LG / -295568,-16718,50800,-23405,30470,-25949,16569,......
LGT / 88,108,-213,-150,-69,-233,-10,-140,.........

In *LGT* the coefficients have been multiplied by a factor 10 and are later divided by the same factor. One can compare the above series and table II.
Another advantage of this new code is the possibility of cross-checking the coefficients as the arrangements of these same coefficients are completely different.
The magnetic field is calculated with this algorithm by the routines **chp45_70** for epochs between 1945-1970, **chp70_95** for epochs between 1970-1995, **chp95_15** for epochs between 1995-2010, **chp95** for epochs greater than 1995, **chp00** for epochs greater than 2000, **chp05** for epochs greater than 2005, **chp10** for epochs greater than 2010 and **chp15** for epochs greater than 2015.

4.7 THE THREE COMPONENTS OF THE GEOMAGNETIC FIELD

The components of the geomagnetic field are given in the $\hat{r}, \hat{\theta}, \hat{\varphi}$ frame of reference:

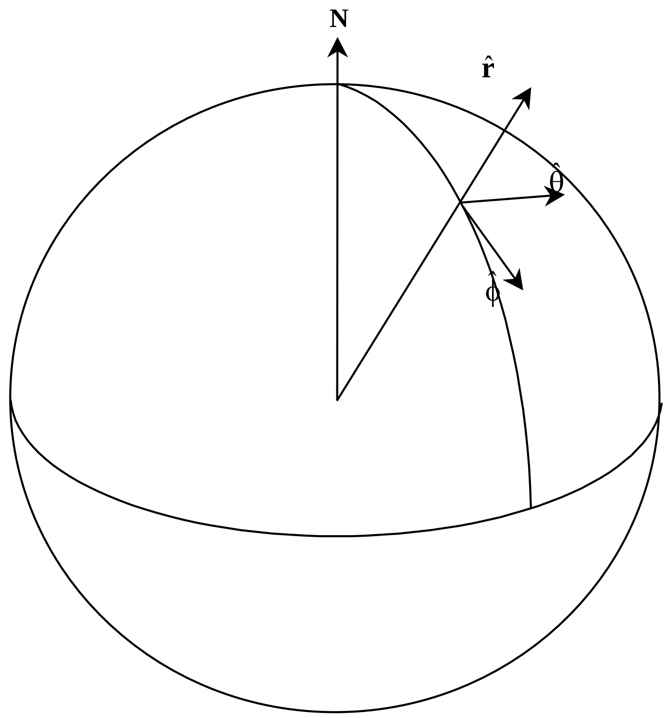

Figure 1

The three components of the geomagnetic field are:

$$
\begin{aligned}
B_r &= -\frac{\partial V}{\partial r} = \sum_{n=1}^{N} \left(\frac{a}{r}\right)^{n+2} (n+1) \sum_{m=0}^{n} \left(g_n^m \cos m\varphi + h_n^m \sin m\varphi\right) P_n^m(\theta) \\
B_\theta &= -\frac{1}{r}\frac{\partial V}{\partial \theta} = -\sum_{n=1}^{N} \left(\frac{a}{r}\right)^{n+2} \sum_{m=0}^{n} \left(g_n^m \cos m\varphi + h_n^m \sin m\varphi\right) \frac{\partial P_n^m(\theta)}{\partial \theta} \qquad (63) \\
B_\varphi &= -\frac{1}{r\sin\theta}\frac{\partial V}{\partial \varphi} = \frac{1}{\sin\theta} \sum_{n=1}^{N} \left(\frac{a}{r}\right)^{n+2} \sum_{m=0}^{n} m\left(g_n^m \sin m\varphi - h_n^m \cos m\varphi\right) P_n^m(\theta)
\end{aligned}
$$

In these formulae B_r is counted positive outwards (in the direction of $\hat{r}$), B_θ is counted positive southwards (in the direction of $\hat{\theta}$), B_φ is counted positive eastwards (in the direction of $\hat{\varphi}$).

4.8 THE TILTED DIPOLE

The first three harmonics of the magnetic field potential can be replaced by a tilted dipole. From (30) we have:

$$V = a\left(\frac{a}{r}\right)^2 \left[g_1^0 \; P_1^0(\theta) + \left(g_1^1 \cos\varphi + h_1^1 \sin\varphi\right) P_1^1(\theta)\right] \qquad (64)$$

where $P_1^0\,(\theta) = \cos\,\theta$ and $P_1^1\,(\theta) = \sin\,\theta$.
We can calculate the magnetic field or the potential produced by a tilted dipole. In the tilted frame of reference we have:

$$V = a\left(\frac{a}{r}\right)^2 \hat{g}_1^0 \cos\theta^* \qquad (65)$$

where $\bar{g}_1^0$ is the magnitude of the tilted dipole and θ^* is the colatitude in the tilted frame of reference. From spherical triangle NPD we obtain readily:

$$\cos\theta^* = \cos\theta_d \cos\theta + \sin\theta_d \sin\theta \cos(\varphi - \varphi_d) \qquad (66)$$

where φ_d is the longitude of the meridian which contains the tilted dipole and θ_d is the tilt.

Introducing (66) into (65) we get:

$$V = a\left(\frac{a}{r}\right)^2 \left[\bar{g}_1^0 \cos\theta_d \cos\theta + \bar{g}_1^0 \left(\sin\theta_d \cos\varphi_d \cos\varphi + \sin\theta_d \sin\varphi_d \sin\varphi\right)\sin\theta\right] \qquad (67)$$

Identity implies the following relationships:

$$\begin{aligned} \bar{g}_1^0 \cos\theta_d &= g_1^0 \\ \bar{g}_1^0 \sin\theta_d \cos\varphi_d &= g_1^1 \\ \bar{g}_1^0 \sin\theta_d \sin\varphi_d &= h_1^1 \end{aligned} \qquad (68)$$

From (68) we obtain:

$$\begin{aligned} \bar{g}_1^0 &= \sqrt{\left(g_1^0\right)^2 + \left(g_1^1\right)^2 + \left(h_1^1\right)^2} \\ \tan\varphi_d &= h_1^1 / g_1^1 \\ \cos\theta_d &= g_1^0 \Big/ \sqrt{\left(g_1^0\right)^2 + \left(g_1^1\right)^2 + \left(h_1^1\right)^2} \end{aligned} \qquad (69)$$

$\bar{g}_1^0$ defines the "strength" of the tilted dipole, θ_d and φ_d define the orientation.

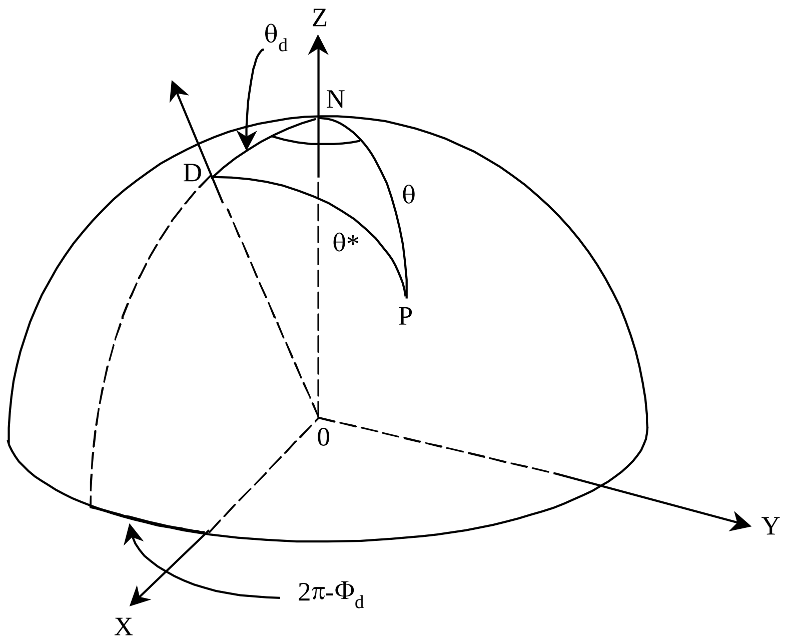

Figure 2

In the software developed for INTERBALL and CLUSTER, the dipole routine contains the first three harmonics and their secular changes. The three magnetic field components of the tilted dipole are:

$$\begin{aligned} B_{rd} &= -2\left(\frac{a}{r}\right)^3 \left[g_1^0 \cos\theta + \left(g_1^1 \cos\varphi + h_1^1 \sin\varphi\right)\sin\theta\right] \\ B_{\theta d} &= \left(\frac{a}{r}\right)^3 \left[-g_1^0 \sin\theta + \left(g_1^1 \cos\varphi + h_1^1 \sin\varphi\right)\cos\theta\right] \\ B_{\varphi d} &= \left(\frac{a}{r}\right)^3 \left[-g_1^1 \sin\varphi + h_1^1 \cos\varphi\right] \end{aligned} \qquad (70)$$

The magnetic field of the tilted dipole is performed in the routine **dipol** for epochs greater than 1995 with the coefficients of DGRF95. The value of the tilted dipole $\bar{g}_1^0$ and its orientation is calculated in the routine **incline.**

4.9 THE ECCENTRIC DIPOLE

In the previous paragraph we have shown that the first three harmonics of the geomagnetic field could be replaced by a tilted dipole. It is possible to show (Schmidt, Ref. 2) that three additional coefficients (g_2^0, h_2^1, g_2^1) of the geomagnetic potential can vanish through a translation of the tilted dipole. A nice solution was obtained (Bernard et al. Ref. 3) using the tools of Quantum Mechanics, namely the Wigner formulae for the rotation of spherical harmonics. Spherical harmonics in Quantum Mechanics can be expressed in the form:

$$Y_n^m(\theta,\varphi) = (-1)^m \sqrt{\frac{2n+1}{4\pi}\frac{(n-m)!}{(n+m)!}}\, P_n^m(\cos\theta)\, e^{im\varphi} \qquad (71)$$

where $(-1)^m$ is a phase factor. Thus

$$P_n^m(\theta)\, e^{im\varphi} = A_n^m\, Y_n^m(\theta,\varphi) \qquad (72)$$

with $A_n^m = a_n^m \sqrt{\dfrac{4\pi}{2n+1}}$

$a_n^m = 1$ if $m = 0$

$a_n^m = 0$ if $m \neq 0$

Taking into account (30) and (71) we get:

$$V = \frac{a}{2}\sum_1^{\infty}\left[\frac{a}{r}\right]^{n+1}\sum_{m=0}^{n}\left\{\left(g_n^m - ih_n^m\right)e^{im\varphi} + \left(g_n^m + ih_n^m\right)e^{-im\varphi}\right\}P_n^m(\theta) \qquad (73)$$

This formula can be compacted by summing between - n and + n. We obtain:

$$V = \frac{a}{2}\sum_1^{\infty}\left[\frac{a}{r}\right]^{n+1}\sum_{m=-n}^{m=+n} B_n^m\left[g_n^m - i\frac{m}{|m|}h_n^m\right]\left[\frac{m}{|m|}\right]^m Y_n^m(\theta,\varphi) \qquad (74)$$

Trough two rotations, the spherical harmonics will be functions of angles Θ, Φ different from angles *θ, φ*. To get the same potential we still have the following equation must be satisfied:

$$\sum_{m=-n}^{m=+n} B_n^m\left[g_n^m - i\frac{m}{|m|}h_n^m\right]\left[\frac{m}{|m|}\right]^m Y_n^m(\theta,\varphi) \equiv \sum_{m=-n}^{m=+n} B_n^m\left[G_n^m - i\frac{m}{|m|}H_n^m\right]\left[\frac{m}{|m|}\right]^m Y_n^m(\Theta,\Phi) \qquad (75)$$

where the G_n^m and H_n^m are the coefficients in the tilted reference system and the Y_n^m (Θ, Φ) are the related spherical harmonics. The relation between the spherical harmonics is obtained with the Wigner formula (Messiah, Ref. 4):

$$Y_n^m(\theta,\varphi) = \sum_{m=-n}^{m=n} R^n_{m',m}(\alpha,\beta,\gamma)\, Y_n^m(\Theta,\Phi) \qquad (76)$$

where $$R^{n}_{m',m}(\alpha,\beta,\gamma) = e^{-im\alpha}\; r^{n}_{m,m}\; e^{-im'\gamma}$$

with $$r^{n}_{m',m} = \sum (-1)^{t} \frac{\left[(n+m')!\,(n-m')!\,(n+m)!\,(n-m)!\right]^{1/2}}{(n+m'-t)!\,(n-m-t)!\,t!\,(t-m'+m)!}\, \xi^{2n+m'-m-2t}\; \eta^{2t-m'+m} \qquad (77)$$

where summation is over 1 + τ terms, where τ is the smallest number between $n \pm m,\ n' \pm m'$. ξ and η are defined as $\eta = \sin\frac{\beta}{2}$, $\xi = \cos\frac{\beta}{2}$, (figure 3).

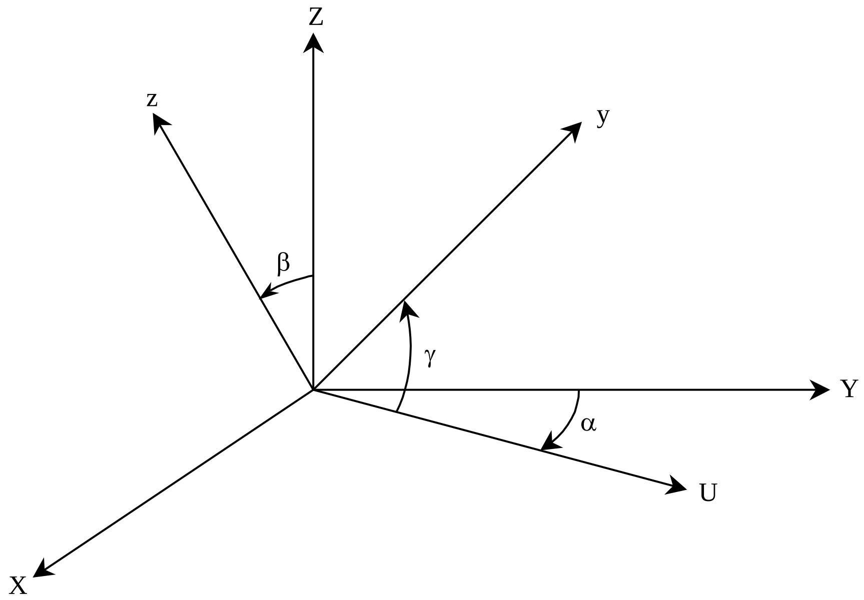

Figure 3

Inserting (76) into (75) and multiplying by the conjugate spherical harmonic function Y_n^{k*} (Θ, Φ) we obtain:

$$B_n^k \left[G_n^k - iH_n^k\right] = \sum_{m=-n}^{m=+n} B_n^m \left[g_n^m - i\frac{m}{|m|} h_n^m \right] \left[\frac{m}{|m|}\right]^m R^n_{k,m} \qquad (78)$$

Taking the conjugate of (78) and performing the addition, we get:

$$2\, B_n^k\, G_n^k = \sum_{m=-n}^{m=+n} B_n^m \left[\frac{m}{|m|}\right]^m \left\{ g_n^m \left(R^n_{k,m} + R^{n*}_{k,m}\right) + ih_n^m \left(R^n_{k,m} - R^{n*}_{k,m}\right)\right\} \qquad (79)$$

where $$R^{n*}_{k,m} = (-1)^{k-m}\, R^{n}_{-k,-m}$$
$$r_{-k,-m} = (-1)^{k-m}\, r_{k,m}$$

Taking into account the symmetry properties we get:

$$G_n^k = \frac{1}{B_n^k} \sum_{m=0}^{n} A_n^m \left\{ g_n^m \left[r_{k,m}^n \cos(k\alpha + m\gamma) + (-1)^m \, r_{k,-m}^n \cos(k\alpha - m\gamma) \right] \right.$$
$$\left. + h_n^m \left[- r_{k,m}^n \sin(k\alpha + m\gamma) - (-1)^m \, r_{k-m}^n \sin(k\alpha - m\gamma) \right] \right\} \qquad (80)$$

In order to obtain the H_n^k we use equation (77) again. Subtracting the complex conjugate to (77) one obtains:

$$H_n^k = \frac{1}{B_n^k} \sum A_n^m \left\{ g_n^m \left[r_{k,m}^n \sin(k\alpha + m\gamma) + (-1)^m \, r_{k,-m}^n \sin(k\alpha - m\gamma) \right] \right.$$
$$\left. + h_n^m \left[- r_{k,m}^n \sin(k\alpha + m\gamma) - (-1)^m \, r_{k-m}^n \sin(k\alpha - m\gamma) \right] \right\} \qquad (81)$$

These general relationships, can be applied to the first three harmonics of the geomagnetic potential. If we set $\alpha = 0$, we get:

$$\begin{aligned} G_1^1 &= g_1^0 \sin\beta + g_1^1 \cos\beta \cos\gamma - h_1^1 \cos\beta \sin\gamma \\ H_1^1 &= g_1^1 \sin\gamma + h_1^1 \cos\gamma \end{aligned} \qquad (82)$$

Angles γ and β can be chosen for avanishing of the coefficients G_1^1 and H_1^1. We obtain:

$$\begin{aligned} \tan\gamma &= -\frac{h_1^1}{g_1^1} \\ \tan\beta &= -\frac{g_1^1 \cos\beta - h_1^1 \sin\gamma}{g_1^0} \end{aligned} \qquad (83)$$

If we compare (69) and (83) we deduce that $\varphi_d = -\gamma$ and $\theta_d = -\beta$. For the three other harmonics, G_2^0, G_2^1, H_2^1 we get:

$$G_2^0 = g_2^0 \left(\cos^2\beta - \frac{1}{2}\sin^2\beta \right) - \sqrt{\frac{3}{4}} \left(g_2^1 \cos\gamma - h_2^1 \sin\gamma \right) \sin 2\beta + \sqrt{\frac{3}{4}} \left(g_2^2 \cos 2\gamma - h_2^2 s \sin 2\gamma \right) \sin^2\beta$$

$$\begin{aligned} G_2^1 &= \sqrt{\frac{3}{4}} \, g_2^0 \sin 2\beta + \cos 2\beta \left(g_2^1 \cos\gamma - h_2^1 \sin\gamma \right) - \frac{1}{2} \sin 2\beta \left(g_2^2 \cos 2\gamma - h_2^2 \sin 2\gamma \right) \\ H_2^1 &= \left(\cos^4\beta - \sin^4\beta \right) \left(g_2^1 \sin\gamma + h_2^1 \cos\gamma \right) - \sin\beta \left(g_2^2 \sin 2\gamma + h_2^2 \cos 2\gamma \right) \end{aligned} \qquad (84)$$

It is possible to calculate the translation of the tilted dipole which will cancel G_2^0, G_2^1, H_2^1. The new dipole remains parallel to the old dipole in this translation (fig. 4).

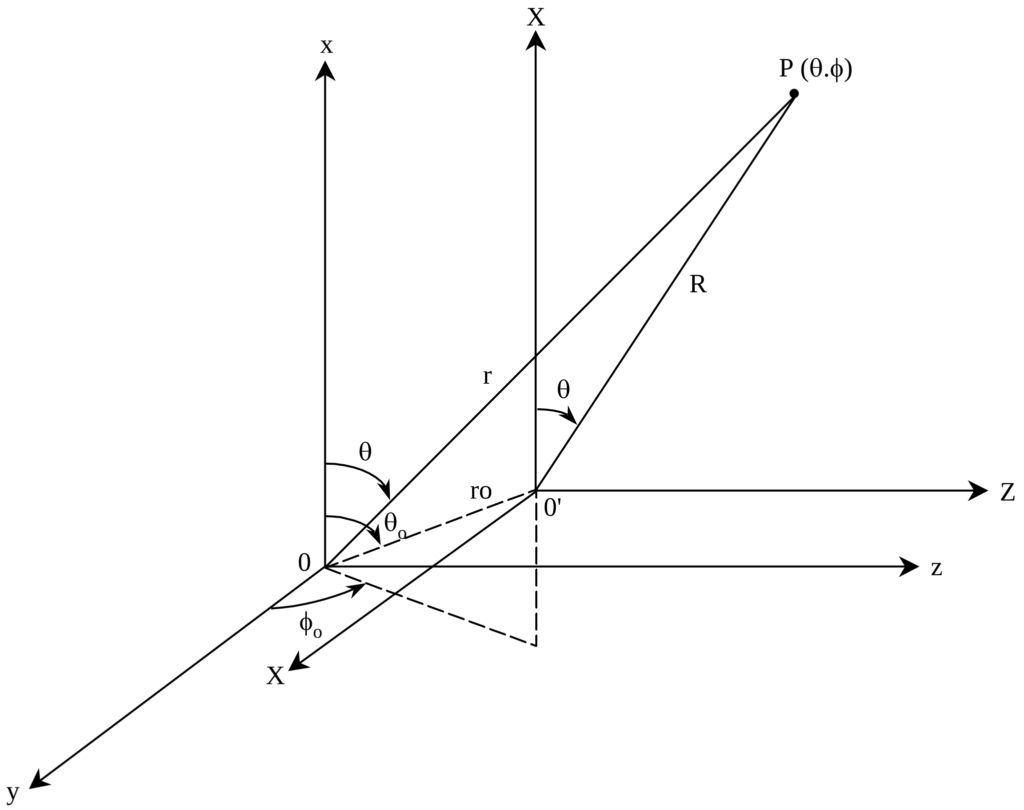

Figure 4

In the old frame of reference a point P is defined by its coordinates (r, θ, φ). In the new frame of reference the point P is defined by the coordinates (R, Θ, Φ). The translation is defined by the vector $\vec{r}_o$ in the direction (θ_o, φ_o). If the translation cancels the three higher harmonics we have:

$$V = a^3 \frac{\overline{G}_1^0}{R^2} P_1^0(\Theta) \equiv \frac{a^3}{r^2} G_1^0 P_1^0(\theta) + \frac{a^4}{r^3}\left[G_2^0 P_2^0(\theta) + \left(G_2^1 \cos\varphi + H_2^1 \sin\varphi\right) P_2^1(\theta)\right] \qquad (85)$$

In this formula $\overline{G}_1^0$ is the coefficient in the eccentric frame of reference. It is possible to calculate R as a function of r and r_o. We get:

$$R^2 = r^2 + r_0^2 - 2rr_0 \cos\psi$$

Where ψ is the diedral angle between vectors $\vec{r}_o$ and $\vec{r}$. From spherical trigonometry it is easy to calculate cos ψ:

$$\cos\psi = \cos\theta_0 \cos\theta + \sin\theta \sin\theta_0 \cos(\varphi - \varphi_{0o})$$

Introducing these two equations in the left member of equation (85) and linearizing we get:

$$
\begin{aligned}
&\overline{G}_1^0 \frac{a^{"}}{r^2}\cos\theta + 3\,\overline{G}_1^0\, a^3 \frac{r_o}{r^{"}}\cos^2\theta\cos\theta_0 + 3\,\overline{G}_1^0\, a^3 \frac{r_o}{r^3}\cos\theta\sin\theta\cos\varphi\cos\varphi_0\sin\varphi_0 \\
&+ 3\,\overline{G}_1^0\, a^{"} \frac{r_0}{r^3}\cos\theta\sin\theta\sin\varphi\sin\theta_0\sin\varphi_0 - \overline{G}_1^0\, a^3 \frac{r_0}{r^3}\cos\theta_0 \\
&\equiv \frac{a^3}{r^{"}} G_1^0\cos\theta + \frac{a^4}{r^3}\left[\frac{1}{2}\left(3\cos^2\theta - 1\right)G_2^0 + \sqrt{3}\sin\theta\cos\theta\left(G_2^1\cos\varphi + H_2^1\sin\varphi\right)\right]
\end{aligned}
\qquad (86)
$$

Identifying members with same trigonometric lines we get:

$$
\overline{G}_1^0 \equiv G_1^0
$$

$$
r_0 = a\sqrt{\frac{4\left({G_2^1}^2 + {H_2^1}^2\right) + 3{G_2^0}^2}{12{G_1^0}^2}}
$$

$$
\cos\theta_0 = \frac{1}{2}\frac{G_2^0}{G_1^0}\frac{1}{\sqrt{\dfrac{4\left({G_2^1}^2 + {H_2^1}^2\right) + 3{G_2^0}^2}{12{G_2^0}^2}}} \qquad (87)
$$

$$
\cos\varphi_0 = \frac{1}{\sqrt{3}}\frac{G_2^1}{G_1^0}\frac{a}{r_0}\frac{1}{\sin\theta_0}
$$

$$
\sin\varphi_0 = \frac{1}{\sqrt{3}}\frac{H_2^1}{G_1^0}\frac{a}{r_0}\frac{1}{\sin\theta_0}
$$

which completely define the translation in the tilted reference frame. The orientation and the displacement of the eccentered dipole are calculated in routine **testdipex2**. This routine uses **incline** for the calculation of the tilted dipole plus two other routines not described in the MAGLIB reference manual (for specialists only).

4.10 CALCULATION OF THE MAGNETIC FIELD COMPONENTS NEAR THE SURFACE OF THE EARTH

If a point P is near the surface of the Earth, it is sometimes interesting to calculate the components of the Earth's magnetic field near the Earth's surface. We calculate the relationship between the geodetic latitude and the geocentric latitude. If a and b are the semi-major axis and the semi-minor axis of the Earth's ellipsoid at the Earth's surface we have:

$$\frac{x^2}{a^2}+\frac{z^2}{b^2}=1 \tag{88}$$

We can derive the components of the normal to the ellipsoid surface using the gradient formula:

$$\vec{\nabla}=\frac{2x}{a^2}\,\hat{i}+\frac{2z}{b^2}\,\hat{k} \tag{89}$$

The normal is defined as $\hat{N}=\vec{\nabla}\Big/\left\|\vec{\nabla}\right\|$

From the following figure we have: $\tan\lambda=\dfrac{\nabla_z}{\nabla_x}=\dfrac{z\,a^2}{x\,b^2}$ (90)

We also have: $z=\rho\sin\beta, x=\rho\cos\beta$ thus $\tan\lambda=\dfrac{a^2}{b^2}\tan\beta$ (91)

From usual trigonometric formulae: $\sin^2\beta=\dfrac{1}{1+\dfrac{1}{\tan^2\beta}}$ (92)

Using (91) we get: $\sin\beta=\dfrac{\sin\lambda}{\left(\sin^2\lambda+\dfrac{a^{'}}{b^4}\cos^2\lambda\right)^{1/2}}$ (93)

We also have $x = a\cos\beta$, $z = b\sin\beta$
we get easily: $\rho^2 = a^2\cos^2\beta + b^2\sin^2\beta$

with: $e^2=1-\dfrac{b^2}{a^2}$ we obtain $\rho^2=a^2\cos^2\beta+a^2\left(1-e^2\right)\sin^2\beta$

thus: $\rho=a\sqrt{1-e^2\sin^2\beta}$

The coordinates of point P are:

$$x = \rho \cos \beta + h \cos \lambda$$
$$z = \rho \sin\beta + h \sin \lambda \quad (94)$$

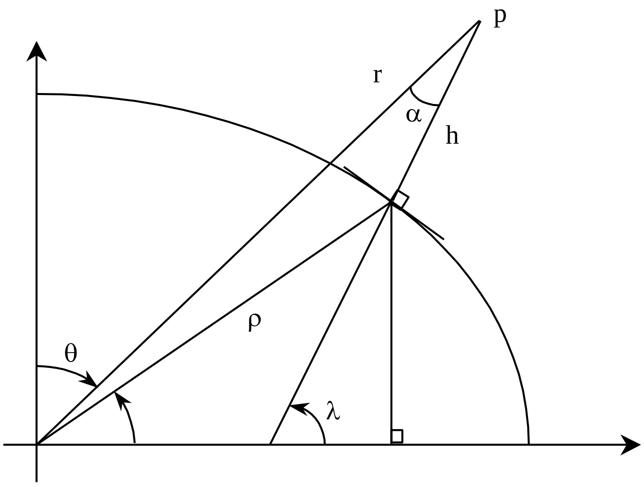

Figure 5

The geocentric distance r and the colatitude θ of point P are:

$$r = \left(x^2 + z^2\right)^{1/2}$$
$$\theta = a\cos\left(\frac{z}{r}\right) \quad (95)$$

The angle α between the local vertical and the geocentric direction is:

$$\alpha = \theta + \lambda - 90° \quad (96)$$

α is positive for λ positive and α is negative for λ negative. It is possible to calculate the vertical and the horizontal components of the Earth magnetic field. If X is the component toward the north, Y the component toward the east and Z the vertical component, we have:

$$X = -B_r \sin\alpha - B_\theta \cos\alpha$$
$$Z = -B_r \cos\alpha + B_\theta \sin\alpha \quad (97)$$
$$Z = B_\varphi$$

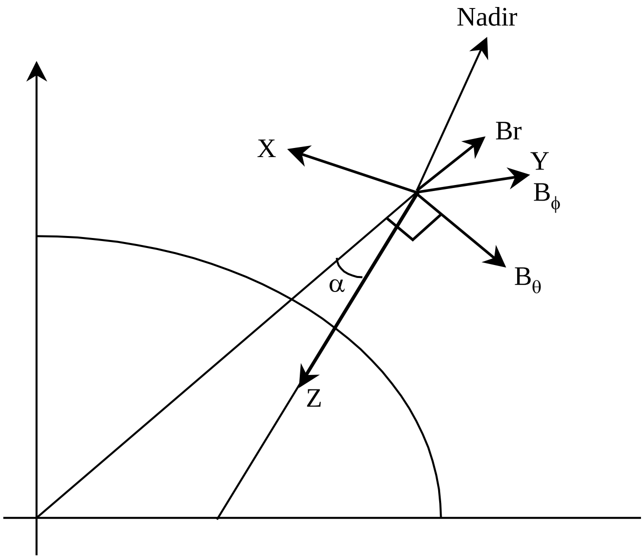

Figure 6

The horizontal component *H* is defined as:

$$H = \sqrt{X^2 + Y^2} \qquad (98)$$

The declination *D* is defined by:

$$\sin D = \frac{Y}{H}, \cos D = \frac{X}{H} \qquad (99)$$

The inclination *I* is defined as:

$$\tan I = \frac{Z}{H} \qquad (100)$$

4.11 DIPOLE MAGNETIC FIELD IN CARTESIAN COORDINATES

For some applications it is useful to calculate the three components of the magnetic field in a cartesian coordinate system. The general expression of the dipole potential is:

$$V = g_1^0 \, \frac{\cos\theta}{r^2} \qquad (101)$$

where θ is the colatitude and r is the radial distance. In a cartesian coordinate system with axis *z* along the dipole axis, the colatitude θ can be expressed as $\theta = a\cos(z/r)$

V can be rewritten as:

$$V = g_1^0 \, z r^{-3} \qquad (102)$$

Taking the partial derivatives: $\left(-\frac{\partial V}{\partial x}, -\frac{\partial V}{\partial y}, -\frac{\partial V}{\partial z} \right)$

We obtain the three components of the magnetic field in a cartesian coordinate system :

$$
\begin{aligned}
B_x &= -3\, g_1^0 \; x r^{-5} \\
B_y &= -3\, g_1^0 \; y r^{-5} \\
B_z &= g_1^0 \; r^{-3} \left(3z^2 \; r^{-5} - 1 \right)
\end{aligned}
\qquad (103)
$$

The dipole magnetic field is calculated in a rectangular coordinate system in the routine **dipols.**

TABLES OF INTERNAL MAGNETIC FIELD COEFFICIENTS

The following tables give the coefficients for the internal magnetic field since epoch 1945 in two forms, the original way compacting the g and the h in one matrix in the software, and our new approach which involves two matrices in the software for the g and the h. The more up-to-date coefficients are provisory and are provided with their secular variations.
For sake of simplicity we have provided not up-to-date modules with their secular variations. For example if a new set of coefficients is given for the year N, the software for the module which corresponds to year N-5 will receive definitive coefficients, but no secular variations. In this case these secular variations are calculated, using interpolation between the set for year N and the set for year N-5. These secular variations are provisory. At year N+5, these secular variations will be definitive as the set of Schmidt coefficients for year N will be definitive.
For example in 2010, Schmidt coefficients for the interim geomagnetic field will be given as well as the secular variations. Schmidt coefficients for 2005 Epoch will be definitive. The secular variations are not given but can be calculated using the sets of 2010 and 2005. In 2015 the coefficients of 2010 will be definitive, thus the calculation of the secular variations for 2005 will give definitive secular variations.
For sake of simplicity the routine which involve a multiple set of coefficients will be updated when a new set of definitive coefficients will be available. For example modules igrf95-05 will be transformed in dgrf95-05 in 2010 as the coefficients for epoch 2005 will be definitive. We suggest to update this routine again in 2015 adding the set of definitive coefficients corresponding to epoch 2010. To dgrf95-05 will correspond chp95-05 in 2010.

CHP15 (Interim coefficients)

```
data lg/
-294420,-15010,47971,-24451,30129,-28456,16767,-6419,13507,-23523,-1153,12256,2449,
5820,-5384,9076,8137,2833,1204,-1887,-3349,1809,704,-3295,-2326,3601,473,
1924,1970,-1409,-1193,-1575,160,41,1002,700,677,-208,727,332,-1299,
589,-289,-667,132,73,-709,626,816,-761,-541,-68,-195,518,57,
150,244,94,34,-28,-274,68,-22,242,88,101,-169,-183,-32,
133,-206,-146,134,162,117,57,-159,-91,-20,21,54,88,-216,
31,108,-33,118,7,-68,-133,-69,-1,78,87,10,-91,-40,
-105,84,-19,-63,32,1,-4,5,46,-5,44,18,-79,-7,
-6,21,-42,24,-28,-18,-12,-36,-87,31,-15,-1,-23,20,
20,-7,-8,-11,6,8,-7,-2,2,-22,17,-14,-2,-25,
4,-20,35,-24,-19,-2,-11,4,4,12,19,-8,-22,9,
3,1,7,5,-1,-3,3,-4,2,2,-9,-9,-1,0,
7,0,-9,-9,4,4,5,16,-5,-5,10,-12,-2,-1,
8,4,-1,-1,3,4,1,5,5,-3,-4,-4,-3,-8/
```

CHP15 - SECULAR VARIATION (Interim coefficients)

```
data lgt/
103,181,-266,-87,-33,-274,21,-141,34,-55,82,-7,-4,
-101,18,-7,2,-13,-91,53,41,29,-43,-52,-2,5,6,
-13,17,-1,-12,14,34,39,0,-3,-1,0,-7,-21,21,
-7,-12,2,3,9,16,10,3,-2,8,-5,4,13,-2,
1,-3,-6,-6,-8,1,2,-2,2,0,-3,-6,3,5,
1,-2,5,4,-2,1,-3,-4,3,3,0,0,0,0,
112*0/
```

CHP10 (Definitive coefficients)

```
data lg/
-2949657,-158642,494426,-239606,302634,-270854,166817,-57573,133985,-232654,-16040,
123210,25175,
63373,-53703,91266,80897,28648,16658,-21103,-35683,16446,8940,-30972,-23087,35729,
4458,
20026,18901,-14105,-11806,-16317,-1,-803,10104,7278,6869,-2090,7592,4418,-14140,
6154,-2283,-6626,1310,302,-7809,5540,8044,-7500,-5780,-455,-2120,4524,654,
1400,2496,1046,703,164,-2761,492,-328,2441,821,1084,-1450,-2003,-559,
1183,-1934,-1741,1161,1671,1085,696,-1405,-1074,-354,164,550,945,-2054,
345,1151,-527,1275,313,-714,-1238,-742,-76,797,843,214,-842,-608,
-1008,701,-194,-624,273,89,-10,-107,471,-16,444,245,-722,-33,
-96,213,-395,309,-199,-103,-197,-280,-831,305,-148,13,-203,167,
165,-66,-51,-176,54,85,-79,-39,37,-251,179,-127,12,-211,
75,-194,375,-186,-212,-21,-87,30,27,104,213,-63,-249,95,
49,-11,59,52,0,-39,13,-37,27,21,-86,-77,-23,4,
87,-9,-89,-87,31,30,42,166,-45,-59,108,-114,-31,-7,
78,54,-18,10,38,49,2,44,42,-25,-26,-53,-26,-79 /
```

CHP10 - SECULAR VARIATION (Interim calculated secular variations)

```
data lgt/
10914,17084,-29432,-9808,-2688,-27412,1706,-13234,2170,-5152,9020,-1300,-1370,
-10346,-274,-1012,946,-636,-9236,4466,4386,3288,-3800,-3956,-346,562,544,
-1572,1598,30,-248,1134,3202,2426,-168,-556,-198,20,-644,-2196,2300,
-528,-1214,-88,20,856,1438,1440,232,-220,740,-450,340,1312,-168,
200,-112,-212,-726,-888,42,376,216,-42,118,-148,-480,346,478,
294,-252,562,358,-102,170,-252,-370,328,308,92,-20,-130,-212,
-70,-142,394,-190,-486,68,-184,104,132,-34,54,-228,-136,416,
-84,278,8,-12,94,-158,-60,314,-22,-68,-8,-130,-136,-74,
72,-6,-50,-138,-162,-154,154,-160,-78,10,-4,-46,-54,66,
70,-8,-58,132,12,-10,18,38,-34,62,-18,-26,-64,-78,
-70,-12,-50,-108,44,2,-46,20,26,32,-46,-34,58,-10,
-38,42,22,-4,-20,18,34,-6,-14,-2,-8,-26,26,-8,
-34,18,-2,-6,18,20,16,-12,-10,18,-16,-12,22,-6,
4,-28,16,-40,-16,-18,16,12,16,-10,-28,26,-8,-2 /
```

CHP05 (Definitive coefficients)

```
data lg/
-2955463, -166905, 507799, -233724, 304769, -25945, 165776, -51543, 13363, -230583,
19886, 124639, 26972, 67251, -52472, 92055, 79796, 28207, 21065, -22523, -37986, 14515,
10000, -30536, -22700, 35441, 4272, 20895, 18025, -13654, -12345, -16805, -1957, -1355,
10385, 736, 6956,  -2033, 7674, 5475, -15134, 6363, -1458, -6353, 1458, 024, -8636, 5094,
7988, -7446, -6114, -165, -2257, 3873, 682, 123, 2535, 937, 1093, 542, -2632, 194, -464, 248,
762, 112, -1173, -2088, -688, 983, -1811, -1971, 1017, 1622, 936, 761, -1125, -1276, -487,
-006, 558, 976, -2011, 358, 1269, -694, 1267, 501, -672, -1076, -816, -125, 81, 876, 292,
-666, -773, -922, 601, -217, -612, 219, 142, 01, -235, 446, -015, 476, 306, -658, 029, -101,
206, -347, 377, -086, -021, -231, -209, -793, 295, -16, 026, -188, 144, 144, 077, -031, -227,
029, 09, -079, -058, 053, -269, 18, -108, 016, -158, 096, -19, 399, -139, -215, -029, -055, 021,
023, 089, 238, -038, -263, 096, 061, -03, 04, 046, 001, -035, 002, -036, 028, 008, -087, -049,
034, -008, 088, -016, -088, -076, 03, 033, 028, 172, -043, -054, 118, -107, -037, -004, 075,
063, -026, 021, 035, 053, -005, 038, 041, -022, -01, -057, -018, -082/
```

CHP05 - SECULAR VARIATION (Definitive calculated secular variations)

```
data lgt/
11612,16526,-26746,-11764,-4270,-22808,2082,-12060,710,-4142,7692,-2858,-3594,-7756,
-2462,-1578,2202,882,-8814,2840,4606,3862,-2120,-872,-774,576,372,-1738,
1752,-902,1078,976,3912,1104,-562,-164,-174,-114,-164,-2114,1988,-418,
-1650,-546,-296,556,1654,892,112,-108,668,-580,274,1302,-56,340,
-78,218,-780,-756,-258,596,272,-78,118,-72,-554,170,258,400,
-246,460,288,98,298,-130,-560,404,266,340,-16,-62,-86,-26,
-236,334,16,-376,-84,-324,148,98,-26,-66,-156,-352,330,-172,
200,46,-24,108,-106,-40,256,50,-2,-64,-122,-128,-124,10,
14,-96,-136,-226,-164,68,-142,-76,20,24,-26,-30,46,42,
22,-40,102,50,-10,0,38,-32,36,-2,-38,-8,-106,-42,
-8,-48,-94,6,16,-64,18,8,30,-50,-50,28,-2,-24,
38,38,12,-2,-8,22,-2,-2,26,2,-56,22,24,-2,
14,-2,-22,2,-6,28,-12,-4,-10,-20,-14,12,-6,6,
-18,16,-22,6,-8,14,12,2,-6,-32,8,-16,6 /
```

CHP00 (Definitive coefficients)

```
data lg/
-296194,-17282,51861,-22677,30684,-24816,16709,-4580,13396, -22880, -2276, 12521,
2934, 7145, -4911, 9323, 7868, 2726, 2500,-2319, -4030, 1198, 1113, -3038, -2188, 3514,
438, 2223, 1719, -1304, -1331,-1686,-393,-129,1063, 723, 682,-174, 742,637, -1609, 651,-59,
-612, 169, 7, -904, 438, 790,-740, -646, 0, -242, 333, 62, 91, 240,69,148,73, -254, -12, -58,
244,66,119, -92,-215, -79,85, -166,-215,91,155,70,89,-79,-149,-70,-21, 50,94,-197,30, 134,
-84, 125,63,-62, -89, -84, -15, 84,93,38,-43,-82,-82,48,-26, -60,17,17,0,-31,40,-5,49,37,-59,
10, -12,20,-29,42,2,3, -22, -11, -74, 27,-17,1,-19,13,15,-9,-1,-26,1, 9,-7,-7,7,-28,17,-9,1,-12,
12, -19, 40, -9,-22,-3, -4, 2, 3, 9, 25, -2,-26,9,7,-5,3,3,0,-3,0, -4,3,-1,-9,-2,-4,-4,8,-2,-9, -
9,3,2,1, 18, -4, -4,13,-10,-4, -1,7,7,-4, 3, 3, 6,-1,3,4, -2,0,-5,1,-9/
```

CHP00 - SECULAR VARIATION (Definitive calculated secular variations)

```
data lgt/
1295, 1183, -2162, -1391, -414, -2258, -263, -1149, -066, -357, 575, -114, -474, -84, -672,
-235, 223, 189, -787, 133, 463, 507, -226, -031, -164, 06, -022, -267, 167, -123, 193, 011,
395, -013, -049, 026, 027, -059, 051, -179, 191, -029, -174, -047, -046, -009, 081, 143, 018,
-009, 069, -033, 033, 109, 012, 064, 027, 049, -077, -038, -018, 063, 023, 008, 02, -014, -051,
012, 02, 027, -03, 036, 021, 014, 047, -026, -067, 043, 043, 041, 012, 007, -008, 012, -014,
029, 003, -026, -01, -037, 005, 005, -006, -011, -018, -047, 009, -02, 024, 009, -002, 01, -006,
002, 015, 009, 007, -003, -013, -014, -014, 004, 001, -011, -009, -021, -01, -002, -02, -011,
005, 002, 003, 000, 003, -001, 003, -004, 007, 004, 000, -002, 002, -003, 002, 002, -004, 001,
-008, -005, 000, 000, -01, 001, 000, -003, 000, -001, 000, -002, -004, -001, 001, -002, 004,
002, 003, 000, -001, 000, 001, 000, 004, 001, -006, 001, 006, 002, 001, 000, 003, 000, 003,
004, -002, -001, -003, -002, -001, 001, 001, 001, -001, 003, -002, 001, -001, 001, 002, 000,
000, -002, -001, -006, 002/
```

CHP95 (Definitive coefficients)

```
data lg/
-296920 -17840, 53060, -22000, 30700, -23660, 16810, -4130, 13350,-22670, -2620, 12490,
3020, 7590, -4270, 9400, 7800, 2620, 2900, -2360, -4180, 970, 1220, -3060, -2140, 3520,
460, 2350, 1650, -1180, -1430, -1660, -550, -170, 1070, 680, 670, -170, 680, 720, -1700, 670,
-10, -580, 190, 10, -930, 360, 770, -720, -690, 10, -250, 280, 40, 50, 240, 40, 170, 80, -240,
-20, -60, 250, 60, 110, -60, -210, -90, 80, -140, -230, 90, 150, 60, 110, -50, -160, -70, -40, 40,
90, -200, 30, 150, -100, 120, 80, -60, -80, -80, -10, 80, 100, 50, -20, -80, -80, 30, -30, -60, 10,
20, 0, -40, 40, -10, 50, 40, -50, 20, -10, 20, -20, 50, 10, 10, -20, 0, -70,75*0/
```

CHP95 - SECULAR VARIATION (Definitive calculated secular variations)

```
data lgt/
1452,1116,-2398,-1354,-32,-2312,-202,-900,92,-420,    688,62,-172,-890,-1282,-154,136,212,
-800, 82, 300, 56, -214, 44, -96,-12,-44,-254,138,-248, 198, -52, 314, 82, -14, 86, 24, -8, 124,
-166, 182, -38, -98, -64, -42, -6, 52, 156, 40, -40, 88, -20, 16, 106, 44, 82, 0, 58, -44, -14, -28,
16, 4, -12, 12, 18, -64, -10, 22, 10, -52, 30, 2, 10, 20, -42, -58, 22, 0, 38, 20, 8, 6, 0, -32, 32,
10, -34, -4, -18, -8, -10, 8, -14, -24, -46, -4, -4, 36, 8, 0, 14, -6, 0, 18, 0, 10, -2, -6, -18, -20, -4,
0, -18, -16, -16, -14, -4, -22, -8, 54, -34, 2, -38, 26, 30, -18, -2, -52,2, 18, -14, -14, 14, -56, 34,
-18, 2, -24, 24, -38, 80, -18, -44, -6, -8, 4, 6, 18, 50, -4, -52, 18, 14, -10, 6, 6, 0, -6, 0, -8, 6,
-2,-18, -4, -8, -8, 16, -4, -18, -18, 6, 4, 2, 36, -8, -8, 26, -20, -8, -2, 14, 14,-8, 6, 6, 12, -2, 6, 8,
-4, 0, -10, 2, -18/
```

CHP90 (Definitive coefficients)

```
data lg/
-29775,-1848,5406,-2131,3059,-2279,1686,-373,1314,-2239, -284, 1248, 293, 802,-352, 939,
780, 247, 325, -240, -423, 84, 141, -299,-214, 353, 46, 245, 154, -109, -153, -165, -69, -36,
97, 61, 65, -16, 59, 82, -178, 69, 3, -52, 18, 1, -96, 24, 77, -64, -80, 2, -26, 26, 0, -1, 21, 5, 17,
9, -23, 0, -4, 23, 5, 10, -1, -19, -10, 6, -12, -22, 3, 12, 4, 12, 2, -16, -6, -10, 4, 9, -20, 1, 15, -12,
11, 9, -7, -4, -7, -2, 9, 7, 8, 1, -7, -6, 2, -3, -4, 2, 2, 1, -5, 3, -2, 6, 4, -4, 3, 0, 1, -2, 3, 3, 3, -1, 0,
-6/
```

CHP85 (Definitive coefficients)

```
data lg/
-29873,-1905, 5500,-2072, 3044,-2197, 1687, -306, 1296,-2208, -310, 1247, 284, 829, -297,
936, 780, 232, 361, -249, -424, 69, 170, -297, -214, 355, 47, 253, 150, -93, -154, -164, -75,
-46, 95, 53, 65, -16, 51, 88, -185, 69, 4, -48, 16, -1, -102, 21, 74, -62, -83, 3, -27, 24, -2, -6,
20, 4, 17, 10, -23, 0, -7, 21, 6, 8, 0, -19, -11, 5, -9, -23, 4, 11, 4, 14, 4, -15, -4, -11, 5, 10, -21,
1, 15, -12, 9, 9, -6, -3, -6, -1, 9, 7, 9, 1, -7, -5, 2, -4, -4, 1, 3, 0, -5, 3, -2, 6, 5, -4, 3, 0, 1, -1, 2,
4, 3, 0, 0, -6/
```

CHP80 (Definitive coefficients)

```
data lg/
-29992, -1956, 5604, -1997, 3027, -2129, 1663, -200, 1281, -2180, -336, 1251, 271, 833,-252,
938, 782, 212, 398, -257, -419, 53, 199, -297, -218, 357, 46, 261, 150, -74, -151, -162, -78-48,
92, 48, 66, -15, 42, 93, -192, 71, 4, -43, 14, -2, -108, 17, 72, -59, -82, 2, -27, 21, -5, -12, 16, 1,
18,11,-23, -2,-10, 18, 6, 7, 0, -18, -11, 4, -7, -22, 4, 9, 3, 16, 6, -13, -1, -15, 5, 10, -21, 1, 16,
-12, 9, 9, -5, -3, -6,-1, 9, 7, 10, 2, -6, -5, 2, -4, -4, 1, 2, 0, -5, 3, -2, 6, 5, -4, 3, 0, 1, -1, 2, 4, 3,
0, 0, -6/
```

CHP75 (Definitive coefficients)

```
data lg/
-30100,-2013, 5675,-1902, 3010,-2067,1632, -68,1276,-2144, -333, 1260, 262, 830, -223,
946, 791, 191, 438, -265, -405, 39, 216, -288, -218, 356, 31, 264, 148, -59, -152, -159, -83,
-49, 88, 45, 66, -13, 28, 99, -198, 75, 1, -41, 6, -4,-111, 11, 71, -56, -77, 1, -26, 16, -5, -14, 10,
0, 22, 12, -23, -5, -12, 14, 6, 6, -1, -16, -12, 4, -8, -19, 4, 6, 0, 18, 10, -10, 1, -17, 7, 10, -21, 2,
16, -12, 7, 10, -4,-1,-5,-1, 10, 4,11, 1, -3, -2, 1, -3, -3, 1, 2, 1, -5, 3, -2, 4, 5, -4, 4, -1, 1, -1, 0,
3, 3, 1, -1, -5/
```

CHP70 (Definitive coefficients)

```
data lg/
-30220, -2068, 5737, -1781,3000,-2047,1611, 25,1287, -2091, -366, 1278, 251, 838, -196,
952, 800, 167, 461, -266, -395, 26, 234, -279, -216, 359, 26, 262, 139, -42, -139, -160, -91,
-56, 83, 43, 64, -12, 15, 100, -212, 72, 2, -37, 3, -6,-112, 1, 72, -57, -70, 1, -27, 14, -4, -22, 8,
-2, 23, 13, -23, -2, -11, 14, 6, 7, -2, -15, -13, 6, -3, -17, 5, 6, 0, 21, 11, -6, 3, -16, 8, 10, -21, 2,
16, -12, 6, 10,-4, -1, -5, 0, 10, 3, 11, 1, -2, -1, 1, -3, -3, 1, 2, 1, -5, 3, -1, 4, 6, -4, 4, 0, 1, -1, 0,
3, 3, 1, -1, -4/
```

CHP65 (Definitive coefficients)

```
data lg/
-30334,-2119, 5776,-1662, 2997,-2016, 1594, 114, 1297,-2038, -404, 1292, 240, 856, -165,
957, 804, 148, 479, -269, -390, 13, 252, -269, -219, 358, 19, 254, 128, -31, -126, -157, -97,
-62, 81, 45, 61, -11, 8, 100, -228, 68, 4, -32, 1, -8, -111, -7, 75, -57, -61, 4, -27, 13, -2, -26, 6,
-6, 26, 13, -23, 1, -12, 13, 5, 7, -4, -12, -14, 9, 0, -16, 8, 4, -1, 24, 11, -3, 4, -17, 8, 10, -22, 2,
15, -13, 7, 10, -4, -1, -5, -1, 10, 5, 10, 1, -4, -2, 1, -2, -3, 2, 2, 1, -5, 2, -2, 6, 4, -4, 4, 0, 0, -2, 2,
3, 2, 0, 0, -6/
```

CHP60 (Definitive coefficients)

```
data lg/
-30421,-2169, 5791,-1555, 3002,-1967, 1590, 206, 1302,-1992, -414, 1289, 224, 878, -130,
957, 800, 135, 504, -278, -394, 3, 269, -255, -222, 362, 16, 242, 125, -26, -117, -156, -114,
-63, 81, 46, 58, -10, 1, 99, -237, 60, -1, -20, -2, -11, -113, -17, 67, -56, -55, 5, -28, 15, -6, -32,
7, -7, 23, 17, -18, 8, -17, 15, 6, 11, -4, -14, -11, 7, 2, -18, 10, 4, -5, 23, 10, 1, 8, -20, 4, 6, -18,
0, 12, -9, 2, 1, 0, 4, -3, -1, 9, -2, 8, 3, 0, -1, 5, 1, -3, 4, 4, 1, 0, 0, -1, 2, 4, -5, 6, 1, 1, -1, -1, 6, 2,
0, 0, -7/
```

CHP55 (Definitive coefficients)

```
data lg/
-30500,-2215, 5820,-1440, 3003,-1898, 1581, 291, 1302,-1944, -462, 1288, 216, 882, -83,
958, 796, 133, 510, -274, -397, -23, 290, -230, -229, 360, 15, 230, 110, -23, -98, -152, -121,
-69, 78, 47, 57, -9, 3, 96, -247, 48, -8, -16, 7, -12, -107, -24, 65, -56, -50, 2, -24, 10, -4, -32, 8,
-11, 28, 9, -20, 18, -18, 11, 9, 10, -6, -15, -14, 5, 6, -23, 10, 3, -7, 23, 6, -4, 9, -13, 4, 9, -11, -4,
12, -5, 7, 2, 6, 4, -2, 1, 10, 2, 7, 2, -6, 5, 5, -3, -5, -4, -1, 0, 2, -8, -3, -2, 7, -4, 4, 1, -2, -3, 6, 7,
-2, -1, 0, -3/
```

CHP50 (Definitive coefficients)

```
data lg/
-30554,-2250, 5815,-1341, 2998,-1810, 1576, 381, 1297,-1889, -476, 1274, 206, 896, -46,
954, 792, 136, 528, -278, -408, -37, 303, -210, -240, 349, 3, 211, 103, -20, -87, -147, -122,
-76, 80, 54, 57, -1, 4, 99, -247, 33, -16, -12, 12, -12, -105, -30, 65, -55, -35, 2, -17, 1, 0, -40,
10, -7, 36, 5, -18, 19, -16, 22, 15, 5, -4, -22, -1, 0, 11, -21, 15, -8, -13, 17, 5, -4, -1, -17, 3, -7,
-24, -1, 19, -25, 12, 10, 2, 5, 2, -5, 8, -2, 8, 3, -11, 8, -7, -8, 4, 13, -1, -2, 13, -10, -4, 2, 4, -3,
12, 6, 3, -3, 2, 6, 10, 11, 3, 8/
```

CHP45 (Definitive coefficients)

```
data lg/
-30594,-2285, 5810,-1244, 2990,-1702, 1578, 477, 1282,-1834, -499, 1255, 186, 913, -11,
944, 776, 144, 544, -276, -421, -55, 304, -178, -253, 346, -12, 194, 95, -20, -67, -142, -119,
-82, 82, 59, 57, 6, 6, 100, -246, 16, -25, -9, 21, -16, -104, -39, 70, -40, -45, 0, -18, 0, 2, -29, 6,
-10, 28, 15, -17, 29, -22, 13, 7, 12, -8, -21, -5, -12, 9, -7, 7, 2, -10, 18, 7, 3, 2, -11, 5, -21, -27,
1, 17, -11, 29, 3, -9, 16, 4, -3, 9, -4, 6, -3, 1, -4, 8, -3, 11, 5, 1, 1, 2, -20, -5, -1, -1, -6, 8, 6, -1,
-4, -3, -2, 5, 0, -2, -2/
```

IGRF15 (Interim coefficients)

```
data lg/
10,-294420,-24451,13507,9076,-2326,700,816,242,54,-19,31,-19,0,
47971,-15010,30129,-23523,8137,3601,677,-761,88,88,-63,-15,-2,-9,
-28456,-6419,16767,12256,1204,1924,727,-68,-169,31,1,-23,4,4,
-1153,2449,-5384,5820,-3349,-1409,-1299,518,-32,-33,5,20,12,5,
2833,-1887,1809,-3295,704,-1575,-289,150,-206,7,-5,-8,-8,-5,
473,1970,-1193,160,1002,41,132,94,134,-133,18,6,9,10,
-208,332,589,-667,73,626,-709,-28,117,-1,-7,-7,1,-2,
-541,-195,57,244,34,-274,-22,68,-159,87,21,2,5,8,
101,-183,133,-146,162,57,-91,21,-20,-91,24,17,-3,-1,
-216,108,118,-68,-69,78,10,-40,84,-105,-18,-2,-4,3,
32,-4,46,44,-79,-6,-42,-28,-12,-87,-36,4,2,1,
-1,20,-7,-11,8,-2,-22,-14,-25,-20,-24,35,-9,5,
-11,4,19,-22,3,7,-1,3,2,-9,-1,7,0,-4,
-9,4,16,-5,-12,-1,4,-1,4,5,-3,-4,-8,-3/
```

IGRF15 - SECULAR VARIATION (Interim coefficients)

```
data lgt/
10,103,-87,34,-7,-2,-3,3,2,0,0,0,0,0,
-266,181,-33,-55,2,5,-1,-2,0,0,0,0,0,0,
-274,-141,21,-7,-91,-13,-7,-5,-6,0,0,0,0,0,
82,-4,18,-101,41,-1,21,13,5,0,0,0,0,0,
-13,53,29,-52,-43,14,-12,1,-2,0,0,0,0,0,
6,17,-12,34,0,39,3,-6,4,0,0,0,0,0,
0,-21,-7,2,9,10,16,-8,1,0,0,0,0,0,
8,4,-2,-3,-6,1,-2,2,-4,0,0,0,0,0,
-3,3,1,5,-2,-3,3,0,3,75*0/
```

DGRF10 (Definitive coefficients)

```
data lg/
100,-2949657,-239606,133985,91266,-23087,7278,8044,2441,550,-194,305,-212,-9,
494426,-158642,302634,-232654,80897,35729,6869,-7500,821,945,-624,-148,-21,-89,
-270854,-57573,166817,123210,16658,20026,7592,-455,-1450,345,89,-203,30,31,
-16040,25175,-53703,63373,-35683,-14105,-14140,4524,-559,-527,-107,165,104,42,
28648,-21103,16446,-30972,8940,-16317,-2283,1400,-1934,313,-16,-51,-63,-45,
4458,18901,-11806,-1,10104,-803,1310,1046,1161,-1238,245,54,95,108,
-2090,4418,6154,-6626,302,5540,-7809,164,1085,-76,-33,-79,-11,-31,
-5780,-2120,654,2496,703,-2761,-328,492,-1405,843,213,37,52,78,
1084,-2003,1183,-1741,1671,696,-1074,164,-354,-842,309,179,-39,-18,
-2054,1151,1275,-714,-742,797,214,-608,701,-1008,-103,12,-37,38,
273,-10,471,444,-722,-96,-395,-199,-197,-831,-280,75,21,2,
13,167,-66,-176,85,-39,-251,-127,-211,-194,-186,375,-77,42,
-87,27,213,-249,49,59,0,13,27,-86,-23,87,4,-26,
-87,30,166,-59,-114,-7,54,10,49,44,-25,-53,-79,-26 /
```

IGRF10 - SECULAR VARIATION (Interim calculated secular variations)

```
data lgt/
1000,10914,-9808,2170,-1012,-346,-556,232,-42,-20,8,10,44,18,
-29432,17084,-2688,-5152,946,562,-198,-220,118,-130,-12,-4,2,-2,
-27412,-13234,1706,-1300,-9236,-1572,-644,-450,-480,-70,-158,-54,20,18,
9020,-1370,-274,-10346,4386,30,2300,1312,478,394,314,70,32,16,
-636,4466,3288,-3956,-3800,1134,-1214,200,-252,-486,-68,-58,-34,-10,
544,1598,-248,3202,-168,2426,20,-212,358,-184,-130,12,-10,-16,
20,-2196,-528,-88,856,1440,1438,-888,170,132,-74,18,42,22,
740,340,-168,-112,-726,42,216,376,-370,54,-6,-34,-4,4,
-148,346,294,562,-102,-252,328,92,308,-136,-138,-18,18,16,
-212,-142,-190,68,104,-34,-228,416,278,-84,-154,-64,-6,-16,
94,-60,-22,-8,-136,72,-50,-162,154,-78,-160,-70,-2,16,
-46,66,-8,132,-10,38,62,-26,-78,-12,-108,-50,-26,16,
-46,26,-46,58,-38,22,-20,34,-14,-8,26,-34,-8,-28,
-6,20,-12,18,-12,-6,-28,-40,-18,12,-10,26,-2,-8 /
```

DGRF05 (Definitive coefficients)

```
data lg/
100,-2955463,-233724,13363,92055,-22700,736, 7988,248,558,-217,295,-215,-016,507799,
-166905,304769,-230583,79796,35441,6956,-7446,762,976,-612,-16,-029,-088,-25945,
-51543,165776,124639,21065,20895,7674,-165,-1173,358,142,-188,021,03,-19886,26972,
-52472,67251,-37986,-13654,-15134,3873,-688,-694,-235,144,089,028,28207,-22523,14515,
-30536,10000,-16805,-1458,123,-1811,501,-015,-031,-038,-043,4272,18025,-12345,-1957,
10385,-1355,1458,937,1017,-1076,306,029,096,118,-2033,5475,6363,-6353,024,5094,-8636,
542,936,-125,029,-079,-03,-037,-6114,-2257,682,2535,1093,-2632,-464,194,-1125,876,206,
053,046,075,112,-2088,983,-1971,1622,761,-1276,-006,-487,-666,377,18,-035,-026,-2011,
1269,1267,-672,-816,81,292,-773,601,-922,-021,016,-036,035,219,01,446,476,-658,-101,
-347,-086,-231,-793,-209,096,008,-005,026,144,-077,-227,09,-058,-269,-108,-158,-19,-139,
399,-049,041,-055,023,238,-263,061,04,001,002,028,-087,-034,088,-008,-01,-076,033,172,
-054,-107,-004,063,021,053,038,-022,-057,-082,-018/
```

DGRF05 - SECULAR VARIATION (Definitive calculated secular variations)

```
data lgt/
1000,11612,-11764,710,-1578,-774,-164,112,-78,-16,46,20,6,14,
-26746,16526,-4270,-4142,2202,576,-174,-108,118,-62,-24,24,16,-2,
-22808,-12060,2082,-2858,-8814,-1738,-164,-580,-554,-26,-106,-30,18,2,
7692,-3594,-2462,-7756,4606,-902,1988,1302,258,334,256,42,30,28,
882,2840,3862,-872,-2120,976,-1650,340,-246,-376,-2,-40,-50,-4,
372,1752,1078,3912,-562,1104,-296,218,288,-324,-122,50, -2,-20,
-114,-2114,-418,-546,556,892,1654,-756,298,98,-124,0,38,12,
668,274,-56,-78,-780,-258,272,596,-560,-66,14,-32,12,6,
-72,170,400,460,98,-130,404,340,266,-352,-136,-2,-8,16,
-86,-236,16,-84,148,-26,-156,330,200,-172,-164,-8,-2,6,
108,-40,50,-64,-128,10,-96,-226,68,-76,-142,-42,26,14,
-26,46,22,102,-10,38,36,-38,-106,-8,-94,-48,-56,2,
-64, 8,-50,28,-24,38,-2,22,-2,2,22,-2,24,-32,
-22,-6,-12,-10,-14,-6,-18,-22,-8,12,-6,8,6,-16 /
```

DGRF00 (Definitive coefficients)

```
data lg/
10, -296194, -22677, 13396, 9323, -2188, 723, 790, 244, 50,-26,27,-22,-2, 51861,-17282,
30684, -22880, 7868, 3514,682,-740,66,94,-60, -17,-3, -9, -24816, -4580, 16709, 12521,
2500, 2223,742,0, -92, 30, 17, -19, 2, 3, -2276, 2934, -4911,7145,-4030,-1304,-1609, 333,
-79, -84,-31,15,9,1, 2726, -2319,1198,-3038,1113,-1686,-59,91,-166,63,-5,-1,-2,-4,
438,1719,-1331,-393,1063, -129, 169, 69,91,-89,37,1,9,13, -174,637,651,-612,7,438, -904,
73,70,-15,10,-7,-5,-4, -646, -242, 62, 240, 148, -254,-58,-12,-79,93,20,7,3,7, 119,-215, 85,
-215, 155, 89,-149,-21,-70,-43,42,17, -3,-4, -197, 134,125,-62,-84,84,38,-82,48,-82,3,1,-4,3,
17, 0,40,49,-59,-12,-29,2,-22,-74,-11,12, -1,-1, 1,13, -9,-26,9,-7,-28,-9,-12,-19,-9,40,-2,4, -4,
3,25,-26,7,3,0,0,3,-9,-4,8,-4,0, -9,2,18,-4, -10, -1, 7, 3, 6, 3,-2,-5,-9,1/
```

DGRF00 - SECULAR VARIATION (Definitive calculated secular variations)

```
data lgt/
100,1295,-1391,-066,-235,-164,026,018,008,012,009,005,001,001,-2162,1183,-414,-357,
223,06,027,-009,02,007,-002,002,000,000,-2258,-1149,-263,-114,-787,-267,051,-033,-051,
012,-006,000,000,000,575,-474,-672,-84,463,-123,191,109,02,029,015,-001,000,004,189,133,
507,-031,-226,011,-174,064,-03,-026,007,-004,-004,-001,-022,167,193,395,-049,-013,-046,
049,021,-037,-013,004,001,-002,-059,-179,-029,-047,-009,143,081,-038,047,005,-014,-002,
004,001,069,033,012,027,-077,-018,023,063,-067,-011,001,-003,003,001,-014,012,027,036,
014,-026,043,041,043,-047,-009,002,-001,003,-008,-014,003,-01,005,-006,-018,009,024,-02,
-01,001,001,001,01,002,009,-003,-014,004,-011,-021,-002,-011,-02,-005,004,001,003,003,
003,007,000,002,002,-004,-008,000,-01,000,-006,000,-003,-001,-002,-001,-002,002,000,000,
000,001,001,002,006,-002,003,003,-002,-003,-001,001,-001,-002,-001,002,000,-001,002,-006
/
```

DGRF95 (Definitive coefficients)

```
data lg/
10,-296920,-22000,13350,9400,-2140,680,770,250,40,-30,3*0, 53060,-17840,30700, -22670,
7800,3520,670,-720,60,90,-60,3*0, -23660,-4130,16810,12490,2900,2350,680,10, -60, 30,20,
3*0, -2620,3020,-4270,7590,-4180,-1180,-1700,280,-90,-100,-40,3*0, 2620,-2360,970,-3060,
1220,-1660,-10,50,-140,80,-10,3*0, 460,1650,-1430,-550,1070,-170,190, 40, 90, -80, 40, 3*0,
-170, 720,670,-580,10,360,-930,80,60,-10,20,3*0, -690,-250,40,240,170,-240,-60,-20,-50,
100, 20,3*0, 110,-210,80,-230,150,110,-160,-40,-70,-20,50,3*0, -200,150,120,-60,-80,80,
50,-80, 30, -80,10,3*0, 10,0,40,50,-50,-10,-20,10,-20,-70,0,45*0/
```

DGRF95 - SECULAR VARIATION (Definitive calculated secular variations)

```
data lgt/
100, 1452, -1354, 92, -154, -96, 86, 40, -12, 20, 8, 54, -44, -4, -2398, 1116, -32, -420, 136,
-12, 24, -40, 12, 8, 0, -34, -6, -18, -2312, -900, -202, 62, -800, -254, 124, -20, -64, 0, -6, -38,
4, 6, 688, -172, -1282, -890, 300, -248, 182, 106, 22, 32, 18, 30, 18, 2, 212, 82, 456, 44, -214,
-52, -98, 82, -52, -34, 10, -2, -4, -8, -44, 138, 198, 314, -14, 82, -42, 58, 2, -18, -6, 2, 18, 26,
-8, -166, -38, -64, -6, 156, 52, -14, 20, -10, -20, -14, -10, -8, 88, 16, 44, 0, -44, -28, 4, 16, -58,
-14, 0, 14, 6, 14, 18, -10, 10, 30, 10, -42, 22, 38, 0, -46, -16, 34, -6, -8, 6, -32, 10, -4, -8, 8,
-24, -4, 36, -4, -14, 2, -8, 6, 14, 0, 0, -2, -18, -4, -18, -16, -4, -8, -22, 24, -2, -2, 2, 26, -18, -52,
18, -14, -56, -18, -24, -38, -18, 80, -4, 8, -8, 6, 50, -52, 14, 6, 0, 0, 6, -18, -8, 16, -8, 0, -18, 4,
36, -8, -20, -2, 14, 6, 12, 6, -4, -10, -18, 2/
```

DGRF90 (Definitive coefficients)

```
data lg/
1,-29775,-2131, 1314, 939, -214, 61, 77, 23, 4, -3, 5406, -1848, 3059,-2239, 780, 353, 65,
-64, 5, 9, -4, -2279, -373, 1686, 1248, 325, 245, 59, 2, -1, 1, 2, -284, 293, -352, 802, -423,
-109, -178, 26, -10, -12, -5, 247, -240, 84, -299, 141, -165, 3, -1, -12, 9, -2, 46, 154, -153, -69,
97, -36, 18, 5, 3, -4, 4, -16, 82, 69, -52, 1, 24, -96, 9, 4, -2, 3, -80, -26, 0, 21, 17, -23, -4, 0, 2,
7, 1, 10, -19, 6, -22, 12, 12, -16, -10, -6, 1, 3, -20, 15, 11, -7, -7, 9, 8, -7, 2, -6, 3, 2, 1, 3, 6, -4,
0, -2, 3, -1, -6, 0/
```

DGRF85 (Definitive coefficients)

```
data lg/
1, -29873, -2072, 1296, 936, -214, 53, 74, 21, 5, -4, 5500, -1905, 3044,-2208, 780, 355, 65,
-62, 6, 10, -4, -2197, -306, 1687, 1247, 361, 253, 51, 3, 0, 1, 3, -310, 284, -297, 829, -424,
-93, -185, 24, -11, -12, -5, 232, -249, 69, -297, 170, -164, 4, -6, -9, 9, -2, 47, 150, -154, -75,
95, -46, 16, 4, 4, -3, 5, -16, 88, 69, -48, -1, 21, -102, 10, 4, -1, 3, -83, -27, -2, 20, 17, -23, -7, 0,
4, 7, 1, 8, -19, 5, -23, 11, 14, -15, -11, -4, 1, 2, -21, 15, 9, -6, -6, 9, 9, -7, 2, -5, 3, 1, 0, 3, 6, -4,
0, -1, 4, 0, -6, 0/
```

DGRF80 (Definitive coefficients)

```
data lg/
1, -29992, -1997, 1281, 938, -218, 48, 72, 18, 5, -4, 5604, -1956, 3027,-2180, 782, 357, 66,
-59, 6, 10, -4, -2129, -200, 1663, 1251, 398, 261, 42, 2, 0, 1, 2, -336, 271, -252, 833, -419,
-74, -192, 21, -11, -12, -5, 212, -257, 53, -297, 199, -162, 4, -12, -7, 9, -2, 46, 150, -151, -78,
92, -48, 14, 1, 4, -3, 5, -15, 93, 71, -43, -2, 17, -108, 11, 3, -1, 3, -82, -27, -5, 16, 18, -23, -10,
-2, 6, 7, 1, 7, -18, 4, -22, 9, 16, -13, -15, -1, 2, 2, -21, 16, 9, -5, -6, 9, 10, -6, 2, -5, 3, 1, 0, 3, 6,
-4, 0, -1, 4, 0, -6, 0/
```

DGRF75 (Definitive coefficients)

```
data lg/
1,-30100,-1902, 1276, 946, -218, 45, 71, 14, 7, -3, 5675,-2013, 3010,-2144, 791, 356, 66, -56,
6, 10, -3, -2067, -68, 1632, 1260, 438, 264, 28, 1, -1, 2, 2, -333, 262, -223, 830,-405, -59,
-198, 16, -12, -12, -5, 191, -265, 39, -288, 216, -159, 1, -14, -8, 10, -2, 31, 148, -152, -83, 88,
-49, 6, 0, 4, -1, 5, -13, 99, 75, -41, -4, 11, -111, 12, 0, -1, 4, -77, -26, -5, 10, 22, -23, -12, -5,
10, 4, 1, 6, -16, 4, -19, 6, 18, -10, -17, 1, 1, 0, -21,16, 7,-4, -5, 10, 11, -3, 1, -2, 3, 1, 1, 3, 4, -4,
-1, -1, 3, 1,-5, -1/
```

DGRF70 (Definitive coefficients)

```
data lg/
1,-30220,-1781, 1287, 952, -216, 43, 72, 14, 8, -3, 5737, -2068, 3000,-2091, 800, 359, 64,
-57, 6, 10, -3, -2047, 25, 1611, 1278, 461, 262, 15, 1, -2, 2, 2, -366, 251, -196, 838, -395, -42,
-212, 14, -13, -12, -5, 167, -266, 26, -279, 234, -160, 2, -22, -3, 10, -1, 26, 139, -139, -91, 83,
-56, 3, -2, 5, -1, 6, -12, 100, 72, -37, -6, 1, -112, 13, 0, 0, 4, -70, -27, -4, 8, 23, -23, -11, -2, 11,
3, 1, 7, -15, 6, -17, 6, 21, -6, -16, 3, 1, 0, -21, 16, 6, -4, -5, 10, 11, -2, 1, -1, 3, 1, 1, 3, 4, -4, 0,
-1, 3, 1, -4, -1/
```

DGRF65 (Definitive coefficients)

```
data lg/
1,-30334,-1662, 1297, 957, -219, 45, 75, 13, 8, -2, 5776, -2119, 2997,-2038, 804, 358, 61,
-57, 5, 10, -3, -2016, 114, 1594, 1292, 479, 254, 8, 4, -4, 2, 2, -404, 240, -165, 856, -390, -31,
-228, 13, -14, -13, -5, 148, -269, 13, -269, 252, -157, 4, -26, 0, 10, -2, 19, 128, -126, -97, 81,
-62, 1, -6, 8, -1, 4, -11, 100, 68, -32, -8, -7, -111, 13, -1, -1, 4, -61, -27, -2, 6, 26, -23, -12, 1,
11, 5, 0, 7, -12, 9, -16, 4, 24, -3, -17, 4, 1, 2, -22, 15, 7, -4, -5, 10, 10, -4, 1, -2, 2, 2, 1, 2, 6, -4,
0, -2, 3, 0, -6, 0/
```

DGRF60 (Definitive coefficients)

```
data lg/
1,-30421,-1555, 1302, 957, -222, 46, 67, 15, 4, 1, 5791, -2169, 3002,-1992, 800, 362, 58, -56,
6, 6, -3, -1967, 206, 1590, 1289, 504, 242, 1, 5, -4, 0, 4, -414, 224, -130, 878, -394, -26, -237,
15, -11, -9, 0, 135, -278, 3, -255, 269, -156, -1, -32, 2, 1, -1, 16, 125, -117, -114, 81, -63, -2,
-7, 10, 4, 4, -10, 99, 60, -20, -11, -17, -113, 17, -5, -1, 6, -55, -28, -6, 7, 23, -18, -17, 8, 10, -2,
1, 11, -14, 7, -18, 4, 23, 1, -20, 8, 3, -1, -18, 12, 2, 0, -3, 9, 8, 0, 5, -1, 2, 4, 1, 0, 2, -5, 1, -1, 6,
0, -7, 0/
```

DGRF55 (Definitive coefficients)

```
data lg/
1,-30500,-1440, 1302, 958, -229, 47, 65, 11, 4, -3, 5820, -2215, 3003,-1944, 796, 360, 57,
-56, 9, 9, -5, -1898, 291, 1581, 1288, 510, 230, 3, 2, -6, -4, -1, -462, 216, -83, 882, -397, -23,
-247, 10, -14, -5, 2, 133, -274, -23, -230, 290, -152, -8, -32, 6, 2, -3, 15, 110, -98, -121, 78,
-69, 7, -11, 10, 4, 7, -9, 96, 48, -16, -12, -24, -107, 9, -7, 1, 4, -50, -24, -4, 8, 28, -20, -18, 18,
6, 2, -2, 10, -15, 5, -23, 3, 23, -4, -13, 9, 2, 6, -11, 12, 7, 6, -2, 10, 7, -6, 5, 5, -2, -4, 0, -8, -2,
-4, 1, -3, 7, -1, -3, 0/
```

DGRF50 (Definitive coefficients)

```
data lg/
1,-30554,-1341, 1297, 954, -240, 54, 65, 22, 3, -8, 5815, -2250, 2998,-1889, 792, 349, 57,
-55, 15, -7, 4, -1810, 381, 1576, 1274, 528, 211, 4, 2, -4, -1, -1, -476, 206, -46, 896, -408, -20,
-247, 1, -1, -25, 13, 136, -278, -37, -210, 303, -147, -16, -40, 11, 10, -4, 3, 103, -87, -122, 80,
-76, 12, -7, 15, 5, 4, -1, 99, 33, -12, -12, -30, -105, 5, -13, -5, 12, -35, -17, 0, 10, 36, -18, -16,
19, 5, -2, 3, 5, -22, 0, -21, -8, 17, -4, -17, -1, 3, 2, -24, 19, 12, 2, 2, 8, 8, -11, -7, 8, 10, 13, -2,
-10, 2, -3, 6, -3, 6, 11, 8, 3/
```

DGRF45 (Definitive coefficients)

```
data lg/
1,-30594,-1244, 1282, 944, -253, 59, 70, 13, 5, -3, 5810, -2285, 2990,-1834, 776, 346, 57,
-40, 7, -21, 11, -1702, 477, 1578, 1255, 544, 194, 6, 0, -8, 1, 1, -499, 186, -11, 913, -421, -20,
-246, 0, -5, -11, 2, 144, -276, -55, -178, 304, -142, -25, -29, 9, 3, -5, -12, 95, -67, -119, 82,
-82, 21, -10, 7, 16, -1, 6, 100, 16, -9, -16, -39, -104, 15, -10, -3, 8, -45, -18, 2, 6, 28, -17, -22,
29, 7, -4, -1, 12, -21, -12, -7, 2, 18, 3, -11, 2, -3, -3, -27, 17, 29, -9, 4, 9, 6, 1, 8, -4, 5, 5, 1, -20,
-1, -6, 6, -4, -2, 0, -2, -2/
```

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5. EXTERNAL MAGNETIC FIELD MODELS

5.1 INTRODUCTION

Since the early sixties important efforts have been undertaken to model the external magnetic field, i.e., to describe the comet-like topology of the Magnetosphere. The first realistic model of the Magnetosphere was published by Mead (1964). This model which could be summarized as the sum of three terms, dipole + compression term + asymmetry term had a considerable success among the modelers of "the motion of the charged particles in the radiation belt". Ten years later was published the first model bases on magnetic field data, the Mead-Fairfield 1973 model. This model could not properly describe the depletion of the magnetic field in the ring current region nor the magnetic field in the tail region. New models were successively developed by Olson and Pfitzer (1974, 1977). The last version varying with the hour of the day and the season (tilt dependent) remained unpublished. Soon after those models arrived the series of the Tsyganenko models (1987, 1989 Ae and Kp). These three models were tilt dependent, showed a nice Neutral Sheet region, but suffered from a poor ring current description, dayside field line escape and nightside field line flaring. A remedy to these defects was offered by Tsyganenko with his 1996_V1 model which takes into account the interplanetary magnetic field as well as the Field Aligned Currents. Two years later we have developed a simpler model, with not all the features of Tsyganenko 96 but without the defects of the 1989 Tsyganenko models. In the scope of the present database we have retained only the following models:

- The Mead 1964 model
- The Mead-Fairfield 1973 model
- The Tsyganenko 1987 model
- The Tsyganenko 1989 Ae and Kp models
- The Kosik 1998 Kp model
- The Tsyganenko 1996_V1 model[*].

5.2 THE MEAD MODEL

The model developed by Mead in 1964 (Ref. 2) resulted from the determination of the Magnetopause magnetic field by a self consistent calculation (Beard, 1964): the dipole is perpendicular to the Solar Wind direction, the angle of the local tangent to the Magnetopause and the distance to the dipole fulfill the equation:

$$p = 2nmv^2 \cos\psi = B^2 / 8\pi$$

[*] available at NSSDC.

where p is the pressure of the Solar Wind (n being the ion density, m their mass, v their speed). ψ is the angle between the normal to the surface and the velocity vector:

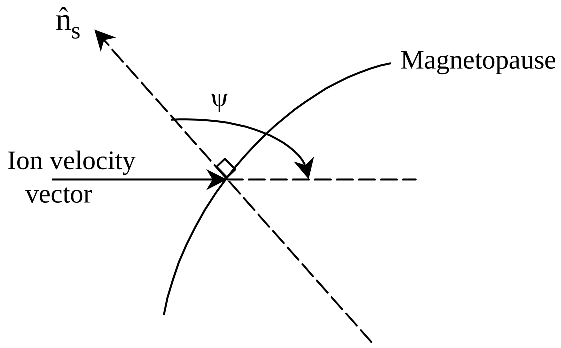

Figure 1

As a result the potential of the magnetic field was developed in a series of spherical harmonics. The development limited to the second order has been widely used:

$$V = a\left\{\frac{a^2}{r^2}g_1^0\cos\theta + \frac{r}{a}\bar{g}_1^0\cos\theta + \frac{\sqrt{3}}{2}\frac{r^2}{a^2}\bar{g}_2^1\sin 2\theta\cos\varphi\right\}$$

In this expression r, θ, φ are the spherical coordinates, θ being the colatitude counted from the north and φ the longitude counted from the midnight meridian. The first term is the dipole magnetic field (g_1^0 = 0.31 gauss) the second term is an axisymmetric compression term ($\bar{g}_1^0$ = 0.00025 gauss) and the third term is the noon-midnight asymmetry term ($\bar{g}_2^1$ = 0.000012 gauss). If we introduce the subsolar distance r_b we can write these coefficients as:

$$\bar{g}_1^0 = \frac{G_S}{r_b^3} \qquad \text{and} \qquad \bar{g}_2^1 = \frac{G_{AS}}{r_b^4} \qquad \text{and we have:}$$

$$V = \frac{g_1^0}{r^2}\cos\theta\left[1+\left(\frac{r}{r_b}\right)^3\frac{G_S}{g_1^0}+\sqrt{3}\left(\frac{r}{r_b}\right)^4\frac{G_{AS}}{g_1^0}\sin\theta\cos\varphi\right]$$

With the change of the subsolar distance the compression and asymmetry terms grow or diminish. This expression enabled the calculation of various magnetospheric effects like diffusion, drift echoes,.....

5.3 THE MEAD-FAIRFIELD MODEL

Mead and Fairfield (Ref. 3) used magnetic field data from several spacecraft (Explorer series 33, 34, 41, 43). The whole set of data extends over 4340 hours of measurements. More than 4.10^7 measurements where used to produce 12616 average values per 0.5 Re box. Data was binned into four classes (Kp = 0^-,0 ,0^+), quiet (Kp <2), perturbed (2< Kp < 3) and highly perturbed (Kp > 3). There was no data for latitudes lower than -50° and for geocentric distances less than -4Re. A least squares fit with Lagrange multipliers was used to get a series of coefficients. The magnetic field model is described as a series of polynomials and is tilt dependent. The tilt parameter T is expressed in units of tens of degrees and the coordinates *X*, *Y*, *Z* must be expressed in units of tens of earth radii:

$$B_X = a_1 Z + a_2 XZ + T\left(a_3 + a_4 X + a_5 X^2 + a_6 Y^2 + a_7 Z^2\right)$$
$$B_Y = b_1 YZ + T\left(b_2 Y + b_3 XY\right)$$
$$B_Z = c_1 + c_2 X + c_3 X^2 + c_4 Y^2 + c_5 Z^2 + T\left(c_6 Z + c_7 XZ\right)$$

The coefficients a_i, b_i, c_i are calculated and the magnetic field fulfills the divergence free condition $\vec{\nabla}.\vec{B} = 0$ through a series of constraint equations:

$$a_2 + b_1 + 2c_1 = 0$$
$$a_4 + b_2 + c_6 = 0$$
$$2a_5 + b_3 + c_7 = 0$$

The coefficients a_i, b_i, c_i are given in Table I:

Table I

Coefficient	Kp = 0, 0+	Kp < 2	Kp > 2	Kp > 3
a_1	17.93	21.79	33.16	39.48
a_2	-5.79	-7.03	-6.39	-2.91
a_3	2.98	3.02	4.30	5.17
a_4	- 2.57	-2.99	-3.25	-3.86
a_5	-0.30	-0.62	-0.44	-1.04
a_6	-1.47	-1.22	-1.27	-1.29
a_7	1.05	0.95	0.45	-1.14
b_1	-10.11	-11.84	-16.54	-19.10
b_2	-1.98	-2.57	-3.08	-3.50
b_3	0.09	-0.28	0.22	0.23
c_1	-9.41	-11.96	-19.88	-22.70
c_2	15.07	17.87	20.23	22.90
c_3	13.16	15.88	22.72	26.50
c_4	8.36	9.77	13.23	15.54
c_5	7.95	9.43	11.46	11.00
c_6	4.55	5.57	6.33	7.36
c_7	0.51	1.53	0.67	1.85

In absence of data measurements near the Earth this model poorly describes the ring current region.

5.4 THE TSYGANENKO 87 MODEL

The Tsyganenko 1987 model (Ref. 5) is based on a data set of 36682 points. The external magnetospheric sources taken into account are the ring current, the tail and the Magnetopause currents.
The ring current model is developed in a cylindrical system aligned with the dipole axis:

$$B_\rho = B_{rc}\,\frac{12\rho\zeta}{\left(\rho^2+\zeta^2+4\right)^{5/2}}$$

$$B_\zeta = 4B_{rc}\,\frac{2\zeta^2-\rho^2+8}{\left(\rho^2+\zeta^2+4\right)^{5/2}}$$

where $\rho = \frac{\left(x_{SM}^2 + y_{SM}^2\right)^{1/2}}{R_{RC}}$ and $\zeta = \frac{z_{Sm}}{R_{RC}}$, B_{RC} is a parameter of the model.

The Magnetotail current distribution is based on a continuous series of filaments. Each filament contributes to the magnetic field:

$$\varDelta B = \frac{R/D}{1+\left(R/D\right)^2} = D\,\frac{\left[\left(x-x_o\right)^2+z^2\right]^{1/2}}{\left(x-x_o\right)^2+z^é+D^2}$$

This description avoids singularities. The initial tail field model of Tsyganenko (1982) model was described by these three equations:

$$B_x = \left[\frac{z}{\pi\left(z^2+D^2\right)^{1/2}}\left(B_N - \frac{x_N - x}{S}\varDelta B\right)\times F(x,z) + \frac{\varDelta B}{2\pi S}\,zG(x,z)\right] f(y)$$

$$B_Y = o.$$

$$B_Z = \left[\left(B_N - \frac{x_N - x}{S}\varDelta B\right)\frac{G(x,z)}{2\pi} + \varDelta B\times\left(1 - \frac{\left(z^2+D^2\right)^{1/2}}{S}F(x,z)\right)\right] f(y)$$

where:

$$F(x,z) = a\tan\frac{x_N - x}{\left(z^2 + D^2\right)^{1/2}} - a\tan\frac{x_N - x - S}{\left(z^2 + D^2\right)^{1/2}}$$

$$G(x,z) = Ln\frac{\left(x_N - x\right)^2 + z^2 + D^2}{\left(x_N - x - S\right)^2 + z^2 + D^2}$$

$$f(y) = \left[1 + \left(y/\Delta y\right)^2\right]^{-1}$$

D = 2 Re, $\Delta\gamma$ = 10 Re, S = 20 Re, x_N = -7 Re

In the 1987 model, Tsyganenko has chosen a more complex expression.
The return currents from the tail are simulated by two additional current sheets parallel to the central one and located at *zgsm* = ± 30 Re. Each of these two current sheets caries an eastward current with a density half of the main sheet current density. The mathematical representation of the tail current system is rather complex and we will not give the expressions here. For the Magnetopause or distant magnetic field the magnetic field components are expressed as follows:

$$B_X = e^{x/\Delta x1}\left[a_1 z\cos\psi + a_2\sin\psi\right] + e^{x/\Delta x2}\left[a_3 z\cos\Psi + \left(a_4 + a_5 y^2 + a_6 z^2\right)\sin\psi\right]$$
$$B_Y = e^{x/\Delta x1}\left[b_1 yz\cos\psi + b_2 y\sin\psi\right] + e^{x/\Delta x2}\left[b_3 yz\cos\psi + \left(b_4 y + b_5 y^3 + b_6 yz^2\right)\sin\psi\right]$$
$$B_Z = e^{x/\Delta x1}\left[\left(c_1 + c_2 y^2 + c_3 z^2\right)\cos\psi + c_4 z\sin\psi\right]$$
$$+ e^{x/\Delta x2}\left[\left(c_5 + c_6 y^2 + c_7 z^2\right)\cos\psi + \left(c_8 z + c_9 zy^2 + c_{10} z^3\right)\sin\psi\right]$$

As in the Mead-Fairfield model six additional relations follow from the requirement $\vec{\nabla}.\vec{B} = 0$. The presence of exponentials enable a better representation of the tail region. The general data set has been divided into 6 subsets from Kp = 0 to Kp > 5. The corresponding coefficients are obtained by least square methods. The routine **tsyg87** corresponds to this model.

5.5 THE TSYGANENKO 1989 MODEL

The Tsyganenko 1989 model (Ref. 6) includes several improvements versus the 1987 model. The tail current sheet is warped and its thickness changes along the Sun-Earth and dawn-dusk directions. The geometry of the warped tail current sheet is shown in the figures below:

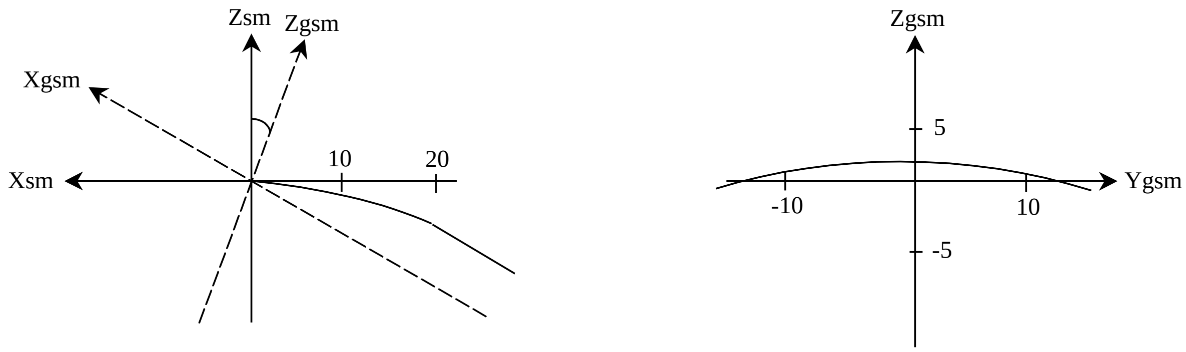

Figure 2

Two versions of the model have been produced:

- The version 1989 Kp for Kp indexes from 0 to 5 (routine **ex89kp**)
- The version 1989 Ae for Ae indexes from 0 to 400 (routine **ex89ae**)

5.6 THE KOSIK 98 MODEL

This model has been developed using poloidal vector fields (Ref. 1). These poloidal vector fields can be expressed in various coordinate systems (cartesian, spherical, cylindrical,...). The poloidal vector fields are divergence free per construction.

$$B_i = \nabla \times \nabla \times (P_i)$$

In this model the magnetic field of the tail is the single non poloidal part of the model. The ring current region is described as a sum of two 0-tilt components and a tilt dependent term. For the 0-tilt components we have:

$$B_r = 2c_{10}\frac{S_1(r)}{r}\cos\theta + 6c_{21}\frac{S_2(r)}{r}\cos\theta\sin\theta\cos\varphi$$

$$B_\theta = -c_{10}\left(\frac{S_1(r)}{r} + \frac{\partial S_1(r)}{\partial r}\right)\sin\theta + \sqrt{3}c_{21}\left(\frac{S_2(r)}{r} + \frac{\partial S_2(r)}{\partial r}\right)\cos 2\theta\cos\varphi$$

$$B_\varphi = -\sqrt{3}c_{21}\left(\frac{S_2(r)}{r} + \frac{\partial S_2(r)}{\partial r}\right)\cos\theta\sin\varphi$$

For the tilt component we have:

$$B_r^T = c_{20} \frac{S_2^T}{r} \left(6\cos^2\theta - 2\right)\sin T$$
$$B_\theta^T = -c_{20} \left(\frac{S_2^T}{r} + \frac{\partial S_2^T}{\partial r} \right) \sin 2\theta \sin T$$
$$B_\varphi^T = 0$$

Where $S_1(r) = r^3 e^{-k_1 r^2}, S_2(r) = r^3 e^{-k_2 r^2}, S_2^T(r) = r^3 e^{-k_2^T r^3}$

T is the tilt angle, the coefficients c_{10} = -1.5, c_{21} = 0.11, k_1 = 0.04, k_2 = 0.01, k_2^T = 0.05 were chosen in order to reproduce the ΔB contours. The coefficient c_{20} can be adjusted. The magnetic field of the distant regions of the Magnetosphere is developed in spherical harmonics:

$$B_r = \sum_{n=1}^{N} \sum_{m=0}^{n} n(n+1) \left(\frac{r}{r_b} \right)^{n-1} a_{nm} \cos m\varphi P_n^m(\theta) \cos T$$
$$B_\theta = \sum_{n=1}^{N} \sum_{m=0}^{n} (n+1) \left(\frac{r}{r_b} \right)^{n-1} a_{nm} \cos m\varphi \frac{\partial P_n^m(\theta)}{\partial \theta} \cos T$$
$$B_\varphi = -\sum_{n=1}^{N} \sum_{m=0}^{n} (n+1) \left(\frac{r}{r_b} \right)^{n-1} a_{nm} \sin m\varphi \frac{m}{\sin\theta} P_n^m(\theta) \cos T$$

coefficients a_{nm} given in Table II are not ichmidt normalized. The return currents are modeled as poloidal vector fields expressed in cylindrical harmonics:

$$B_{c\rho} = \frac{k}{2} (J_0 - J_2) b \sin\lambda e^{kZ}$$
$$B_{c\lambda} = -(J_0 - J_2) b \cos\lambda e^{kZ}$$
$$B_{cZ} = J_1 b \sin\lambda e^{kZ}$$

where the angle λ is counted from the ygsm axis. J_0 and J_2 are the Bessel functions. The coefficient b is given in Table II.
The tail field model has been borrowed from the Tsyganenko and Usmanov 1982 model. We recall their equations:

$$B_{tX} = \left[\frac{z}{\pi\left(z^2 + D^2\right)^{\frac{1}{2}}} \left(B_N - \frac{x_N - x}{S} B_T \right) F(x,z) + \frac{B_T}{2\pi S} z G(x,z) \right] f(y)$$
$$B_{tY} = 0$$
$$B_{tZ} = \left[\left(B_N - \frac{x_N - x}{S} B_T \right) \frac{G(x,z)}{2\pi} + \frac{B_T}{\pi} \left(1 - \frac{\left(z^2 + D^2\right)^{\frac{1}{2}}}{S} F(x,z) \right) \right] f(y)$$

where:

$$F(x,z) = \tan^{-1}\frac{x_N - x}{\left(z^2 + D^2\right)^{\frac{1}{2}}} - \tan^{-1}\frac{x_N - x - S}{\left(z^2 + D^2\right)^{\frac{1}{2}}}$$

$$G(x,z) = Ln\frac{\left(x_N - x\right)^2 + z^2 + D^2}{\left(x_N - x - S\right)^2 + z^2 + D^2}$$

$$f(y) = \left[1 + \left(\frac{y}{\Delta y}\right)^2\right]^{-1} \qquad \text{and} \qquad S = x_N - x_F$$

Typical values are D= 2 Re, $\Delta\gamma$ = 10 Re, x_N = -7 Re, S, B_N, B_r are the parameters given in Table II. The model Kosik98 is tilt and Kp dependent for Kp values between 1 and 5.

TABLE II

iopt	1	2	3	4	5
Kp	1-,1+	2-,2,2+	3-,3,3+	4-,4,4+	5-,5,5+
a_{10}	10.98617	9.91009	12.37934	10.32178	3.03703
a_{21}	14.72302	16.17999	18.53984	62.21787	-0.61821
a_{30}	-3.35283	-4.20870	-4.67247	-63.52502	-0.31021
a_{32}	4.81772	5.52585	6.68590	74.90408	-2.19368
a_{41}	0.89381	0.98896	1.24239	41.8594	-0.74172
a_{41}	-1.71353	-2.40256	-2.91884	-127.25317	-0.56961
a_{50}	0.16678	0.22109	0.20900	27.08902	-0.03409
a_{52}	-0.09963	-0.18175	-0.28215	-41.93544	-0.12372
a_{54}	0.07181	0.07170	0.09359	8.13140	-0.07406
r_b	12.	12.	12.	36.	10.
x_N	-7.	-7.	-7.	-7.	-7.
B_N	50	40	40	40	50
B_r	45.	30.	30.	30.	40.
S	70.	70.	70.	70.	70.
facrc	1.0	1.3	1.5	1.8	2.0
c_{20}	-0.4	-0.4	-0.2666	-0.2666	0.1333
b	25.	30.	35.	50.	60.

This model is more complex than the models Tsyganenko 87 or Tsyganenko 89 but it correctly describes the ring current region. There is no flaring of the field lines in the night side nor a field line escape in the dayside. The computing time is about five times longer with Kosik98 model than with these two models. It is however eight times faster than the new Tsyganenko 96_V1 model which is briefly presented below. The routine **kk97kp** corresponds to this model.

5.7 THE TSYGANENKO 96_V1 MODEL

This model is fairly complete (Ref. 7) and consists of:

- A ring current
- A model of the warped tail
- return currents on the Magnetopause
- Field aligned currents
- Interplanetary magnetic field

The model is based on a least squares fit of the NSSDC data base. The Magnetopause fits the Magnetopause model of Sibeck for different Solar Wind pressures. To achieve these results the variational principle developed by Schulz and Mc Nab has been applied and the magnetic field escape through the Magnetopause is set to zero or near zero. The adjustable parameters are:

Pdyn	between 0.5 and 10 Nanopascals
Dst	between -100 and +20

By and Bx components of the Interplanetary Magnetic field between -10 and + 10 Nanoteslas. This model is not a final version. More information on this model and futures updated versions can be obtained from the author at NSSDC.

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Ref. 6 N.A. Tsyganenko: A Magnetospheric Magnetic Field Model with a Warped Tail Current Sheet, Planet. Sp. Sci., 37, 1, p 5, 1989

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6. CONJUGATE POINT CALCULATIONS

6.1 INTRODUCTION

The determination of conjugate points, points located on the same magnetic field line, implies the calculation of the field line, also called field line tracing. Four types of conjugate point calculations are encountered:

- Conjugacy between a point in space and the Earth (north or southern hemisphere)
- Conjugacy between the two hemispheres
- Conjugacy between two points in space
- Conjugacy between a point in space or a point on the Earth and the geomagnetic equatorial point.

We describe two different algorithms for field line tracing with their respective advantages and drawbacks. Then we stress the difficulties that can be encountered in the tracing with the present magnetospheric models.

6.2 DEFINITION OF THE CONJUGACY

Strictly speaking conjugacy between two points is encountered when the two points are on the same magnetic field line. The strict application of this definition would result in a very limited amount of conjugate phenomena. In past mission analysis we preferred to define the conjugacy of two points as the presence of these two points in the same magnetic flux tube. The width of the flux tube being defined by its extent in invariant latitude and geomagnetic local time.

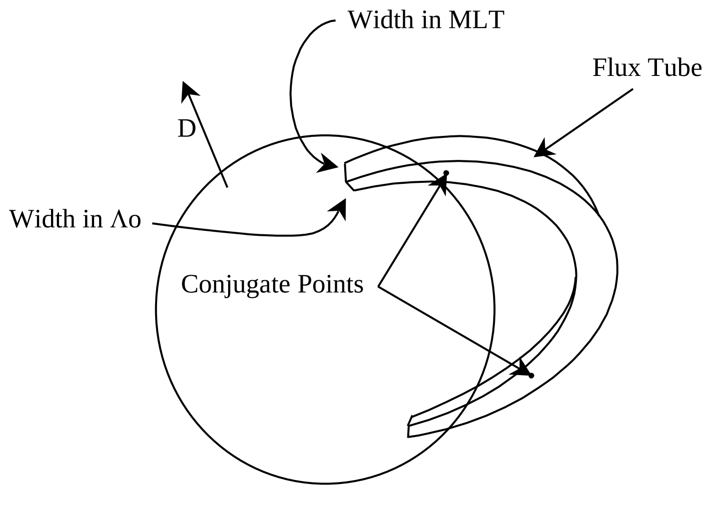

Figure 1

6.3 THE CONJUGACY BETWEEN A POINT IN SPACE AND A POINT ON GROUND

From a point in space the direction of field line tracing depends wether a northern or a southern conjugate point is seeked. For spacecraft located at positions S1 and S2 , the northern conjugate is obtained by a field line integration in the direction of the magnetic field vector B. On the other side, for a southern conjugate point, the integration is opposite to the direction of the magnetic field vector. The same considerations apply for the conjugacy between locations S1 and S2 depending on which starting point is chosen.

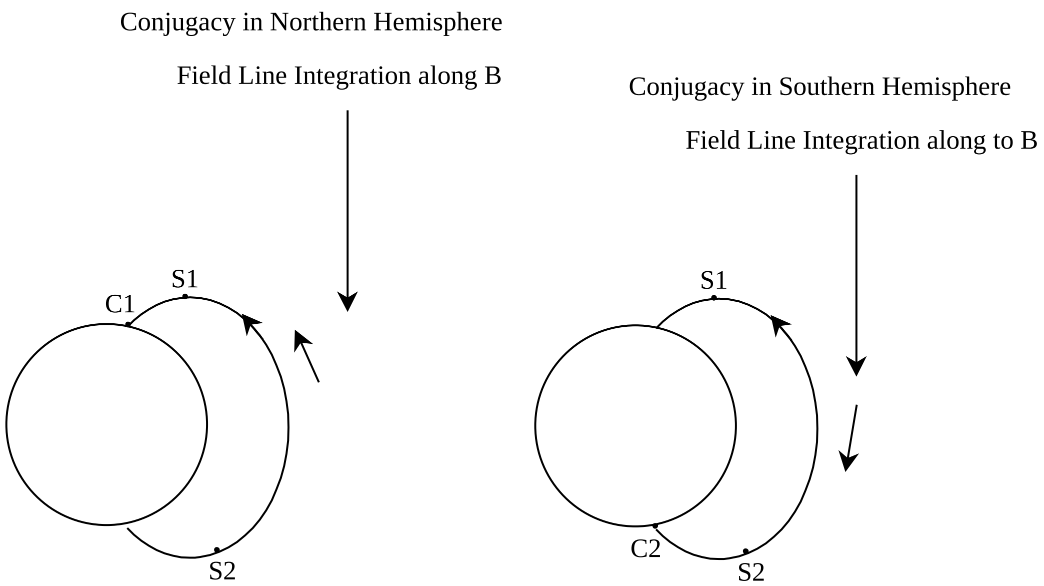

Figure 2

The calculation of conjugate points can be done using routines **pconjr, dconjr,** the last one being more precise, but requiring more computation time.

6.4 EQUATORIAL CONJUGATE

The equatorial conjugate of a ground station or a spacecraft is obtained by field line tracing and corresponds to the point for which the magnetic field B is minimum. As the field line tracing has a finite integration step we do not obtain the true equatorial point but the nearest point of the field line. In the case of an open field line this equatorial point is never obtained. Routine **econjr** calculates the equatorial conjugate.

6.5 THE MERSON ALGORITHM

The Merson algorithm, like the Fehlberg or Dormand Price algorithms belongs to the category of the imbedded Runge Kutta formulae. These algorithms have been discussed in great detail in Ref. 1 and 2. The idea is to construct Runge Kutta formulae which contain and expression $\hat{y}_1$ of higher order than the usual approximation y_1. This expression of $\hat{y}_1$ can be used for the evaluation of the error and for the step size control. The scheme of the coefficients is given as in table I

0					
c_2	a_{21}				
c_3	a_{31}	a_{32}			
c_s	a_{s1}	a_{s2}		$a_{s,s-1}$	
	b_1	b_2		b_{s-1}	$\hat{b}_s$
	$\hat{b}_1$	$\hat{b}_2$		$\hat{b}_{s-1}$	$\hat{b}_s$

The usual Runge Kutta formula of order p is:

$y_1 = y_o + h\,(b_1k_1 + \ldots\ldots\ldots\ldots\ldots\ldots + b_sk_s)$

while

$\hat{y}_1 = y_o + h\left(\hat{b}_1k_1 + \ldots\ldots\ldots\ldots\ldots\ldots + \hat{b}_sk_s\right)$

is of order q (q = p - 1 or q = p + 1)

The error estimate is:

$\hat{y}_1 - y_1$

The table of coefficients for the Merson algorithm is (Ref. 3):

0					
1/3	1/3				
1/3	1/6	1/6			
1/2	1/8	0	3/8		
1	1/2	0	-3/2	2	
	1/2	0	-3/2	2	
	1/6	0	0	2/3	1/6

The Merson algorithm is employed in routine **dconjr.**

6.6 THE ADAMS METHOD

The Adams Moulton method of order 4 is a predictor corrector method. The Adams interpolation formula is

$$y_{n+1} = y_n + \frac{h}{24}\left(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2}\right)$$

The Adams extrapolation formula is:

$$y_{n+1}=y_n+\frac{h}{24}\left(55y_n^{'}-59y_{n-1}^{'}+37y_{n-2}^{'}-9y_{n-3}^{'}\right)$$

Where h is the step of integration. This method is employed in routine **pconjr**

6.7 FIELD LINE TRACING PROBLEMS

In the early sistxies most of the field line tracing was performed using an internal magnetic field model like IGRF. As a consequence all the field lines were closed field lines. The development of more realistic quantitative magnetic field models of the Magnetosphere lead to a more complex topology: field lines are usually closed in the day side and in the night side for low geomagnetic latitudes. At high latitudes field lines are not closed and are very extended. Sometimes, due to the limitations of the models, field lines escape in the dayside of the Magnetosphere near the Cusp regions and wander in the Solar Wind region where no magnetic field model has been built. Thus the calculation of the conjugate point in the other hemisphere is impossible as well the determination of the equatorial conjugate. It is necessary to set an upper limit to the number of points or to the radial distance to stop the field line tracing process. It is also necessary to verify that the field line remains inside the Magnetosphere using for example the Shabansky parabolic Magnetopause or the Sibeck model. For the same reason it is mandatory to verify that the spacecraft or the starting point is inside the Magnetosphere before starting a field line tracing. Just because the field line tracing is time consuming it might be tempting to simultaneously calculate the equatorial conjugate and then the electric field potential and also the length of the field line. Our experience is not in favor of such a method. The main reason is the lack of reliability of the software which must take care of all the impossibilities. If the spacecraft is in the southern hemisphere it is not always possible to obtain the northern conjugate and the equatorial conjugate (Figure 3).

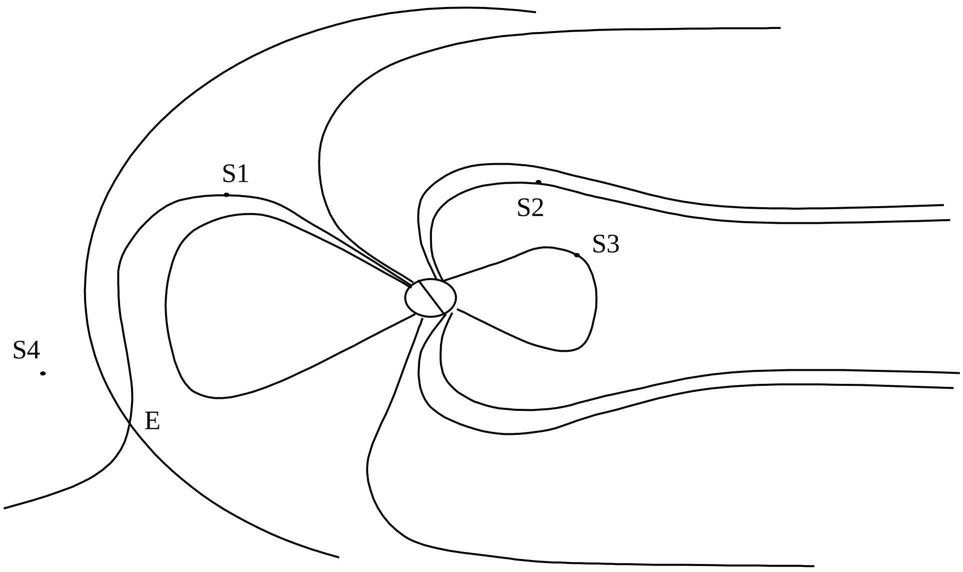

Figure 3

If the spacecraft is in S4 no field line tracing is possible. If it is in S1 field line escape can occur for some models and usually for hight tilt angles. If the spacecraft is in location S2 the northern conjugate can be calculated, thus the invariant latitude the Galperin L and the MLT. But it will not be possible to calculate the equatorial conjugate as well as the southern conjugate. However for location S3 all the calculations can be done. In some cases the Electric field potential cannot be calculated if the location S3 is too far from the Earth. The escape of the field line from the Magnetosphere during the field line tracing is tested using the routine **mpause.**

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7. GEOPHYSICAL PARAMETERS

7.1 INTRODUCTION

In this chapter several important magnetospheric parameters are defined such as the geomagnetic local time, the McIlwain L parameter, The Galperin L parameter, the invariant latitude, the electric field potential of Mc Ilwain. We also define the corrected geomagnetic coordinates. We retain only the definitions of Hakura and Stasiewicz, the definition of Gustafsson involving rather complex calculatons. The definition of the corrected geomagnetic longitude paves the way to the definition of the corrected geomagnetic local time. This corrected geomagnetic local time can be compared to the «magnetic noon» introduced more than thirty years ago by Lebeau.

7.2 THE GEOMAGNETIC LOCAL TIME

The geomagnetic local time of a point is defined as:

$$MLT = \left(\frac{\varphi_D - \varphi_{DO}}{15} \right)_h + 12h$$

Where φ_D is the geomagnetic longitude of the point and φ_{DO} is the geomagnetic longitude of the Sun.

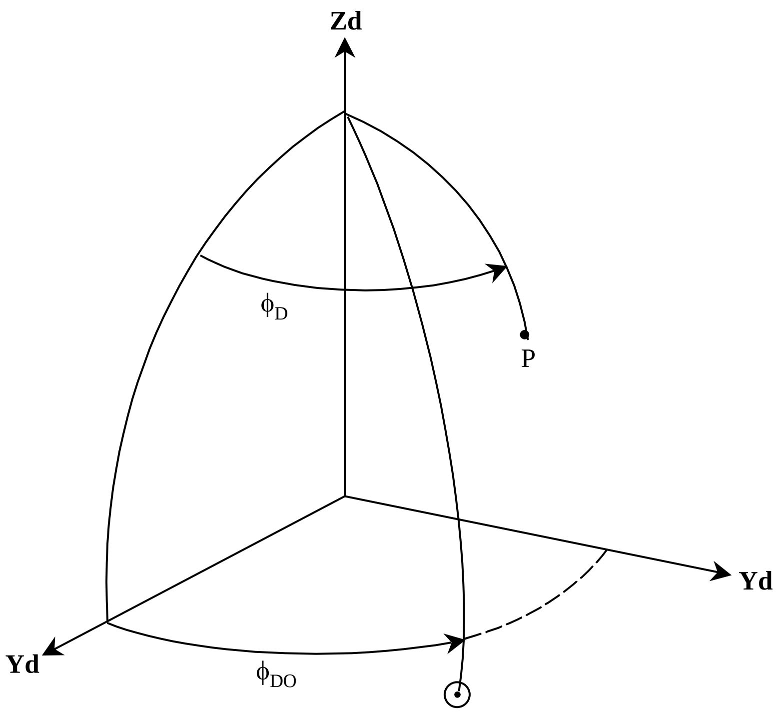

Figure 1

The solar magnetic coordinate system is defined as follows: the axis Zsm is along the dipole and the axis Xsm is such as the plane (Xsm, Zsm) contains the Sun direction. The Ysm axis forms a right handed triedron. The solar magnetic longitude of point P is counted from the (Xsm,Zsm) plane. Thus we have also:

$$\varphi_{SM} = \varphi_D - \varphi_{DO}$$

Another possible definition of MLT is thus:

$$MLT = \left(\frac{\varphi_{SM}}{15}\right)_h + 12h$$

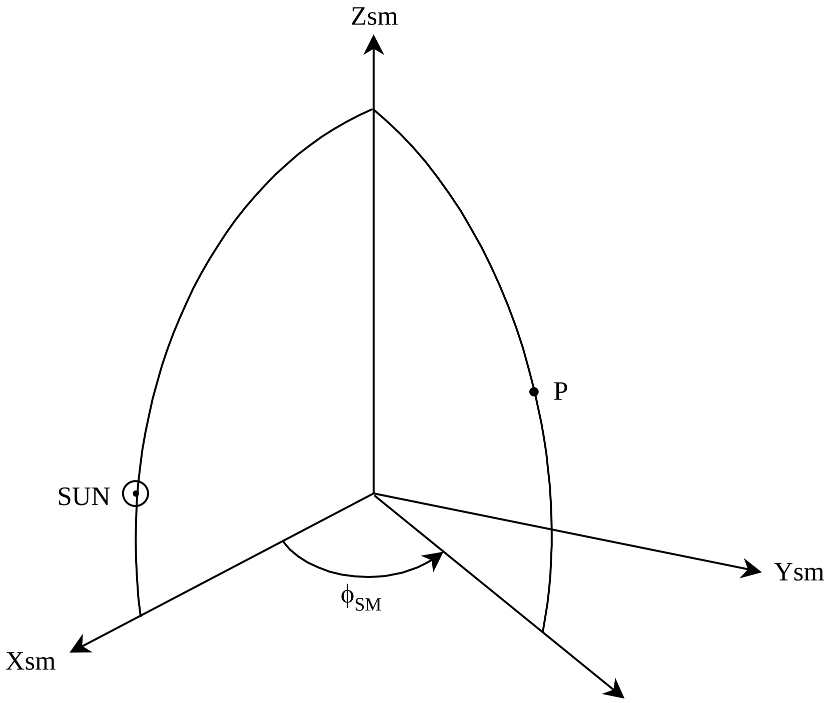

Figure 2

Two routines calculate the geomagnetic local time of a point knowing its geocentric colatitude and longitude (**tgml)** or using the solar magnetic coordinates (**tgml2)**

7.3 THE MC ILWAIN L PARAMETER

The motion of the charged particles in the radiation belt can be described by two invaraiants μ and I defined as:

$$\mu = \frac{E}{B_m} \qquad \text{and} \qquad I = \int_{\theta_m}^{\pi-\theta_m} \sqrt{1 - \frac{B}{B_m}}\, ds$$

Called respectively the first and second invariants. In usual conditions (absence of an electric field and of magnetic field perturbations) the energy E is constant. The first invariant reduces to B_m, the magnetic field intensity at the mirror point. B is the magnetic field intensity at the local point of the field line. The integral is along the field line (half bounce trajectory). Particles with a given B_m and I will drift around the Earth along the same shell. Particles with different B_m and I will drift along different shells. In the case of the Earth , for particles located on the same field line in the radiation belt, the effect is small. Mc Ilwain has found a function $F\left(I^3B/M\right)$ which is almost constant along a line of force. For a given B and a given I it is possible to calculate a shell parameter L which characterizes together with B_m the charged particle population in the radiation Belt. The invariant I is calculated by the subroutine **invar**. For a dipole field the shell parameter L is simply defined by the equatorial point of the magnetic field line: it is equal to the geoecentric distance of the equatorial point expressed in earth radii.

7.4 THE GALPERIN L PARAMETER

For the radiation belt region Mc Ilwain derived the L parameter as a function of the second invariant I and as a function of the local magnetic field. In the outer regions the magnetic field is highly distorted and Y.Galperin suggested the following recipee for a new L parameter:

- Trace the field line from the point down to the Earth with the complete internal+ external magnetic field.
- From the conjugate point (110km altitude), calculate the usual Mc Ilwain L parameter.

One should notice that for the second part of the calculation we still use the magnetic field coefficients and the internal magnetic field model of the sixties.

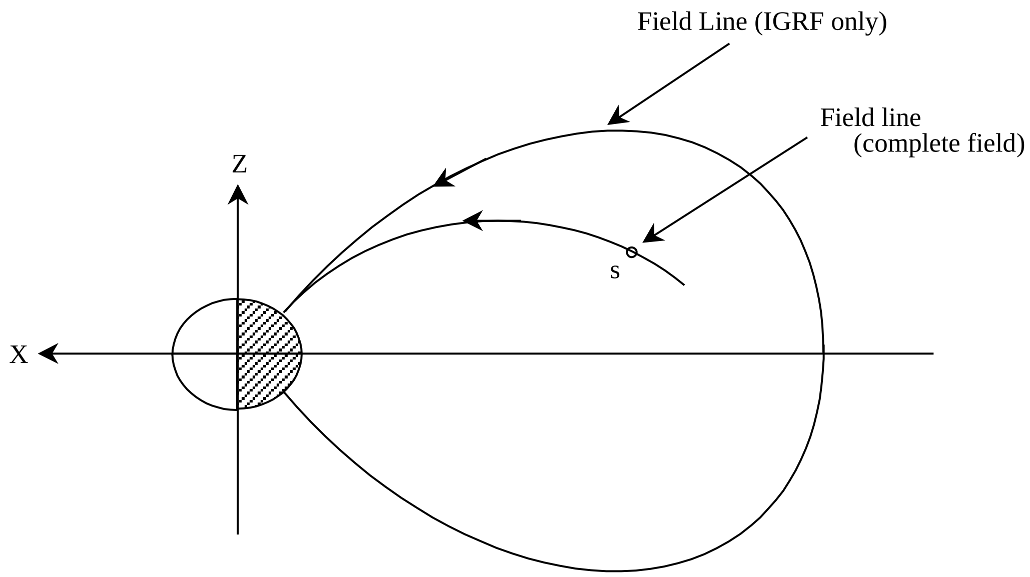

Figure 3

The Galperin L parameter is calculated by the routines **dlgalp** and **flgalp.**

In 2007, Kosik (Ref.6) has described the quantitative aspects of the Galperin L parameter. In some sense, the Galperin *L* parameter can be considered as a geomagnetically corrected McIlwain *L* parameter.

7.5 THE INVARIANT LATITUDE

The invariant latitude is defined as Λ_o :

$$\cos^2 \Lambda_o = \frac{1}{L}$$

Where L is the Galperin L parameter defined in the previous paragraph. The parameter is calculated in **invlat.**

7.6 THE ELECTRIC POTENTIAL

In the equatorial plane, Mc Ilwain (Ref.8) defines a magnetic field model labelled M2 given by the equation:

$$B = 6 - 24\cos\varphi + \frac{18\cos^2\varphi}{1+1728/R^3} + \frac{31000}{R^3}$$

Where φ is the local time and R the radial distance in earth radii, B is expressed in nanoteslas. The electric potential in a non rotating reference frame is given by the equation:

$$\Phi = 10 - 92\left(\frac{B}{31000}\right)^{1/3} + \sum_{i=1}^{6}\sum_{j=1}^{20} A_{ij} \exp\left\{-a_i\left(B - B_i\right)^2 - b_j\left[1-\cos\left(\varphi - \varphi_j\right)\right]\right\}$$

where the coefficients A_{ij}, a_i, b_i, b_j, φ_j are given in Table I. We also have

$$a_i = \frac{Ln2}{d_i^2} \qquad \text{and} \qquad b_j = \frac{Ln2}{1-\cos C_j}$$

B_i and d_i in Table II are in nanoteslas and φ_j, C_j are in hours.

TABLE I

j	A_{1j}	A_{2j}	A_{3j}	A_{4j}	A_{5j}	A_{6j}	φ_j	C_j
1	2.8	5.4	0.6	2.9	-1.2	0.6	4	2
2	6.0	-1.7	1.7	-1.1	0.9	-0.2	6	2
3	-6.5	3.2	-1.2	1.9	-1.1	0.4	6	2
4	5.7	-2.5	1.1	-0.9	0.5	-0.2	10	2
5	-1.4	1.6	-2.2	1.3	-1.2	-0.1	12	2
6	4.5	-3.3	1.0	-2.1	0.5	-0.7	14	2
7	-5.3	0.7	-3.2	0.6	-1.5	-0.1	16	2
8	3.6	-3.8	0.	-1.5	-0.2	-0.7	18	2
9	-3.1	-2.0	-2.5	-0.6	-1.3	-0.4	20	2
10	1.7	-1.5	0.2	-0.9	-0.3	-0.2	21	1
11	1.0	-1.5	-1.1	-0.4	-0.6	-0.2	22	1
12	1.7	-0.5	-0.5	-0.6	0.3	-0.2	22.5	0.5
13	2.9	-2.1	1.7	-1.2	0.4	-0.2	23	0.5
14	-0.1	3.6	-4.9	3.5	-2.2	0.7	23.5	0.5
15	2.7	-3.4	5.5	-0.6	0.6	-0.5	0.	0.5
16	0.7	5.7	2.1	0.6	0.1	-0.2	0.5	0.5
17	6.0	2.3	2.3	0.3	0.1	-0.1	1	0.5
18	3.5	0.3	2.6	-0.8	0.7	-0.4	1.5	0.5
19	9.5	5.1	2.8	1.7	-0.7	0.3	2	1
20	3.0	-1.9	2.1	-1.3	0.9	-0.4	3	1

TABLE II

i	1	2	3	4	5	6
B_i	0	40	100	180	280	400
d_i	30	50	70	90	110^2	130

The Mc Ilwain electric potential is calculated in **mcilwe.**

7.7 THE CORRECTED GEOMAGNETIC COORDINATES

Hultquist (1958a) calculated the spherical harmonic coefficients of the Earth’s magnetic field in a centered dipole coordinate system. In a later work (Hultquist, 1958b) he calculated the deviations of the real field line from the dipole field line due to the perturbations cause by the higher spherical harmonic coefficients. The "integrated deviations" along a dipole field line give displacement vectors in the northern and southern hemispheres. The corrected geomagnetic coordinates of a point are the dipole geomagnetic coordinates corrected from the displacement vector. Hakura (1965) proposed a new method for the calculation of the corrected geomagnetic coordinates:

7.8 HAKURA SOLUTION FOR CORRECTED GEOMAGNETIC COORDINATES

From a point Q on the Earth's surface a field line is traced down to the dipole geomagnetic equator with the complete geomagnetic Earth potential and crosses this equator at point A. From point A a dipole field line is traced down to the Earth (point Qc). This point Qc has the geomagnetic coordinates (θ_c, φ_c) where θ_c is the corrected dipole colatitude and φ_c is the corrected dipole longitude. θ_c is calculated using the geocentric distance of point A:

$$\sin^2 \theta_c = \frac{1}{r_e}$$

where r_e is expressed in earth radii. The corrected dipole longitude φ_c can be calculated from the coordinates of point Qc in the dipole coordinate system.

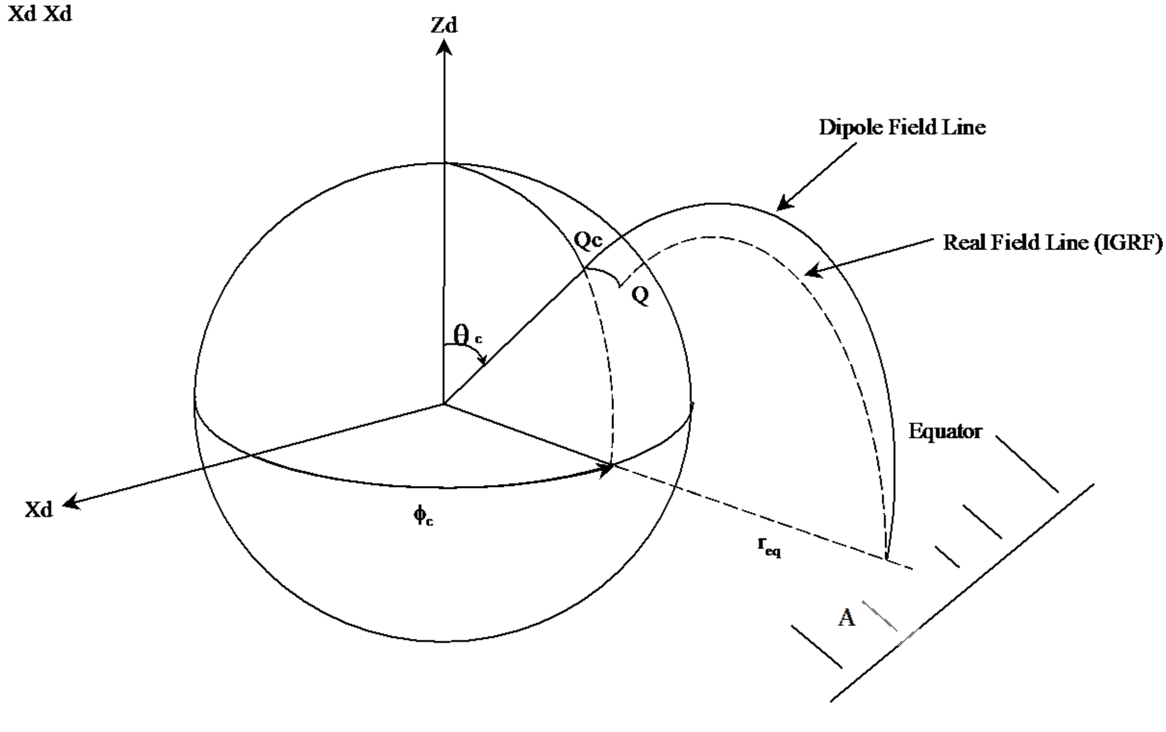

Figure 4

The displacement vectors calculated by Hultquist (1958b) correspond to the projections of the vector QcQ along the meridian and perpendicular to the meridian through point Q.

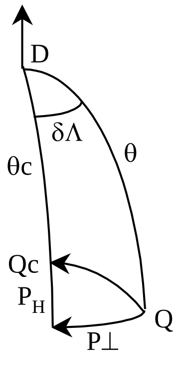

Figure 5

7.9 GUSTAFSSON SOLUTION FOR CORRECTED GEOMAGNETIC COORDINATES

Gustafsson introduces the CBM system.

The CBM system is obtained with the Bmin point on a field line. The distance R_B to the Bmin point of a field line is obtained using the relation:

$$R_B = \left(H_o / B_{\min} \right)^{1/3}$$

The equatorial field intensity is calculated with the first three harmonics. The latitudes are defined by the relation

$$\cos \lambda = \left(R_B \right)^{-1/2}$$

The CBM system introduces the total field in a second step. Field lines are traced to the total field geomagnetic equator and the minimum B locations are marked with their geographic coordinates. An origin meridian is chosen. The advantage of this corrected geomagnetic system lies in the Bmin concept which corresponds to the motion of charged particles which drift in the equatorial surface.

7.10 STASIEWICZ SOLUTION FOR CORRECTED GEOMAGNETIC COORDINATES

Hakura obtained corrected geomagnetic coordinates using the magnetic field of the Earth. Stasiewicz (Ref. 12) extended his results using internal and external magnetic fields. The recipe is the following:

- From a point P in space a field line is traced down to the Earth using the total magnetic field (IGRF + Tsyganenko 87 or 89)
- A point Q on Earth is obtained. At this stage the method of Hakura is used. From this point a field line is traced down to the geomagnetic equator using the internal magnetic field only (IGRF). A point A is obtained on the geomagnetic equator.
- From the point A a dipole field line is traced back to the Earth. Point Qc gives the corrected geomagnetic coordinates.

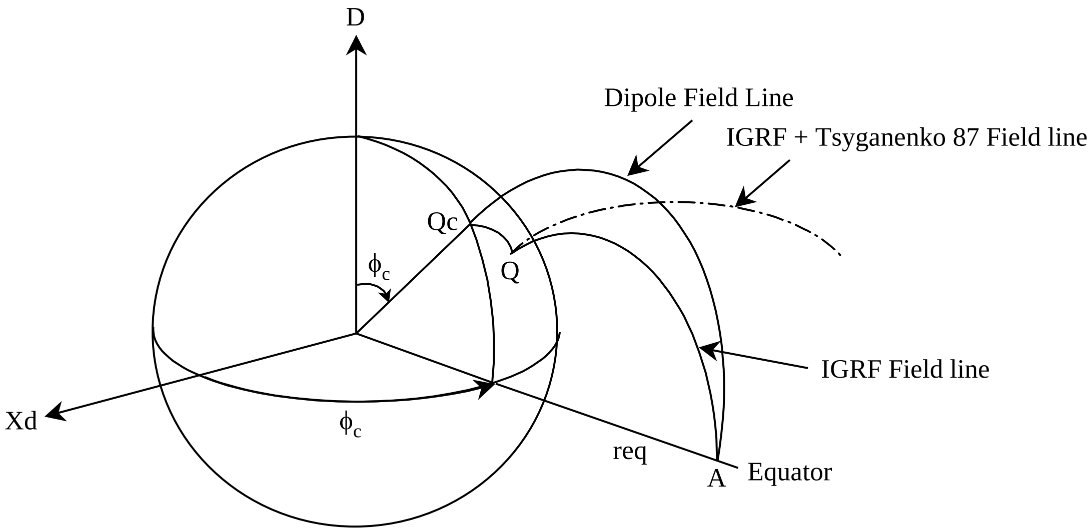

Figure 6

The corrected geomagnetic coordinates are calculated in **corgm.**

7.11 THE CORRECTED GEOMAGNETIC LOCAL TIME

In 1965 Lebeau (Ref. 7) defined the "True Magnetic Noon" as follows:

- For a given epoch A and a given universal time T it is possible to calculate the geographic location of the subsolar point:

$$\lambda_S = -15\,(T - 12)$$
$$\varphi_S = \varepsilon \sin 2\pi A$$

The first equation indicates that the subsolar point is at longitude 0 at Noon. The second equation indicates that the latitude of the subsolar point depends on the season and is null for the equinoxes.
It is also possible to calculate the geomagnetic location (in the dipole coordinate system) of the subsolar point. The geomagnetic longitude Λ_S of the subsolar point can be calculated for time T at epoch A.
For all the field lines of geomagnetic longitude Λ_S it is "Magnetic Noon" at time T for epoch A. It is true in particular for all the points of a real field line which starts at a great distance from the Earth (>6Re) Lebeau traces the dipole field lines of colatitudes 1, 2, 3, 5, 7, 10. They cut a sphere of 10 Re radius. From these intersections he traces the real field line back to the Earth down to 100km. altitude. This calculation is performed for 24 values of T. He obtains "Iso-Magnetic-Noon" curves, which converge towards the "Invariant Pole". There are two "Invariant Poles", one for the northern hemisphere and the other one for the southern hemisphere.

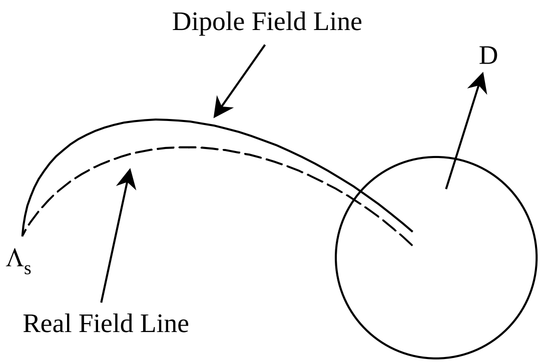

Figure 7

This early definition of the geomagnetic time for the description of auroral phenomenon can be put in perspective with the following definition of the corrected geomagnetic local time by Stasiewicz.

The usual geomagnetic local time of a point is defined as:

$$MLT = \frac{(\varphi_d - \varphi_{do})}{15} + 12h$$

Where φ_d is the geomagnetic longitude of the point and φ_{do} is the geomagnetic longitude of the Sun.
The corrected geomagnetic local time MLT_c will be defined as:

$$MLT_c = \frac{(\varphi_{dc} - \varphi_{do})}{15} + 12h$$

where φ_{dc} is the corrected geomagnetic longitude of the point.
In the Hakura approximation, this corrected geomagnetic local time corresponds to the definition by Lebeau of the "Magnetic Noon".

7.12 APPLICATIONS OF THE GALPERIN L PARAMETER

In the outer magnetosphere directionnal fluxes are adequately described by the Galperin L parameter as is shown in the following figure: a bunch of particles (arrow) measured on the equator will have a dipole L parameter of 5.2,. As this bunch of particles bounces along the real field line the dipole L value will change to 6 and mirror for L = 7. The use of the L dipole for labelling directionnal fluxes induces a continuum of L values between 5.2 and 7. If one uses the Galperin L parameter we have a unique label for the directionnal flux along the distorted field line : the Galperin L parameter is obtained by tracing the real field line to the Earth, than trace back to the equator. Here the Galperin L value equals 7 and remains constant for this bunch of particles along its bounce motion.

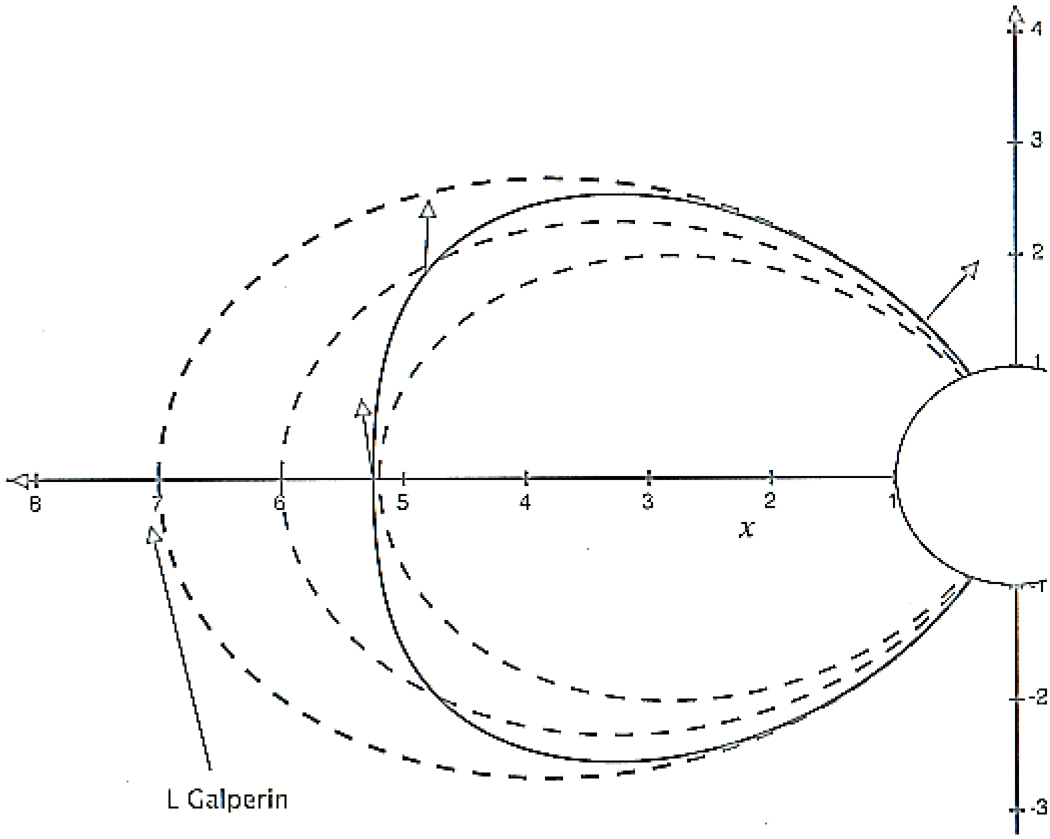

Figure 8

The Galperin L parameter has been successfully used by McIlwain and Kerr (Ref. 10) for the association of Cluster data with auroral displays. As mentionned by these authors another advantage of the Galperin L parameter is the possible labelling of open field lines as one traces the total magnetic field line down to the Earth and then one calculates the McIlwain L value with the internal field only.

There is also a link between the Galperin L parameter and mathematical models of the magnetosphere. As an example we consider the very simple Mead (1964) magnetic field model of the magnetosphere where we retain only the first three harmonics (Kosik, Ref. 5) :

$$V = \frac{1}{r^2}\, g_1^0 \cos\theta + r\, \bar{g}_1^0 \cos\theta + \frac{\sqrt{3}}{2}\, r^2\, \bar{g}_2^1 \sin 2\theta \cos\varphi$$

In this expression r, θ, φ are the spherical coordinates, r is the radial distance expressed in Earth radii, θ is the colatitude counted from the north and φ the longitude counted from the noon meridian. The first term is the dipole magnetic field (g_1^0 = - 0.31 gauss) the second term is an axisymmetric compression term ($\bar{g}_1^0$ = - 0.00025 gauss) and the third term is the noon-midnight asymmetry term ($\bar{g}_2^1$ =- 0.000012 gauss). It was possible to calculate the equations of the field lines using the perturbation theory (Kosik, Ref. 5):

$$r = L \sin^2\theta \left\{ 1 - \frac{1}{2}\frac{\bar{g}_1^0}{g_1^0} L^3 \sin^2\theta + \frac{3}{4}\left(\frac{\bar{g}_1^0}{g_1^0}\right)^2 L^6 \sin^{12}\theta + 2\sqrt{3}\,\frac{\bar{g}_2^1}{g_1^0} L^4 \left(\frac{\sin^4\theta}{7} - \frac{\sin^9\theta}{3}\right)\cos\phi_0 \right\}$$

$$\phi = \phi_0 + \frac{\sqrt{3}}{7} \frac{\bar{g}_2^{-1}}{g_1^0} L^4 \sin^7\theta \sin\phi_0$$

In the following picture we have traced a field line of the model and its associated dipole field line which has the same L value:

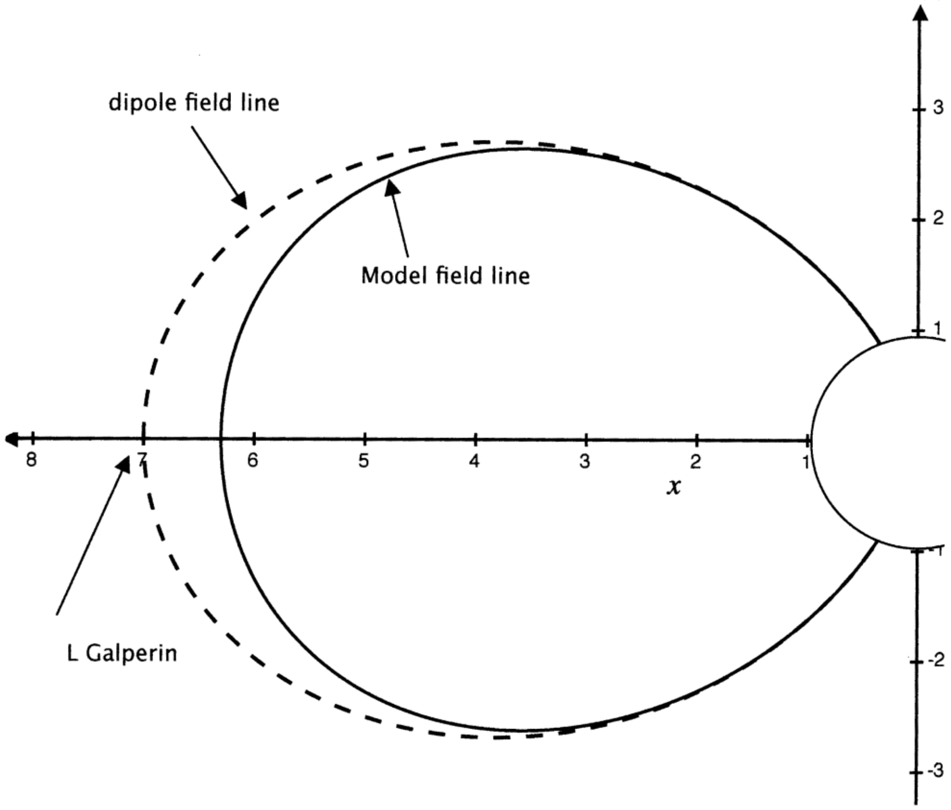

Figure 9

A bunch of particles will follow the model field line and the directionnal flux and according to the Galperin definition will have the label L of its associated dipole field line. From the field line equations above we notice that the Galperin L parameter is indeed the L parameter in the above equations. There is a one to one correspondence between the Galperin L and the mathematical description of the model. As a consequence all the mathematical results for the description of the particle motion or the fluxes can also be viewed in terms of the galperin L parameter. For example the drift shell equation appears as a relation between Galperin L parameters of different longitudes:

$$L = L_0 \left[1 + \frac{\bar{g}_2^{-1}}{g_1^0} L_0^4 \, p(\theta_m) (\cos\phi - \cos\phi_0) \right]$$

where L is the Galperin L parameter at longitude ϕ and L_0 is the Galperin L parameter at longitude ϕ_0. In this equation $p(\theta_m)$ is a function of the miror point colatitude θ_m of the particle.

These examples show the possibilities offered by the Galperin L parameter for the labelling of fluxes and the modelling of charged particles motion.

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8. ASTRONOMY AND CELESTIAL MECHANICS

8.1 INTRODUCTION

Mission Analysis and Space Science data treatment require orbital calculations as well as various astronomical informations such as the Inertial and Greenwich frames of reference, the Sun position and motion, sideral time,.... Due to the attraction of the Sun, the Moon and the planets the inertial frame of reference is not exactly inertial, the Ecliptic changes slowly as well as the Celestial Pole. In the present chapter a short review of all this material is made. For further details a small bibliography is given at the end of this chapter.

8.2 FRAME OF REFERENCES

<u>The Azimuth-elevation coordinate system</u>:

In this system the observer is at the origin of the coordinate system. The fundamental plane is the local horizon and the Z axis is the normal to the free surface of a liquid. In this coordinate system a direction is defined by two angles:

- The azimuth, A, counted positive from the local north in the clockwise sense.
- The elevation, h, counted positive towards the Zenith from the local horizon and negative towards the Nadir.

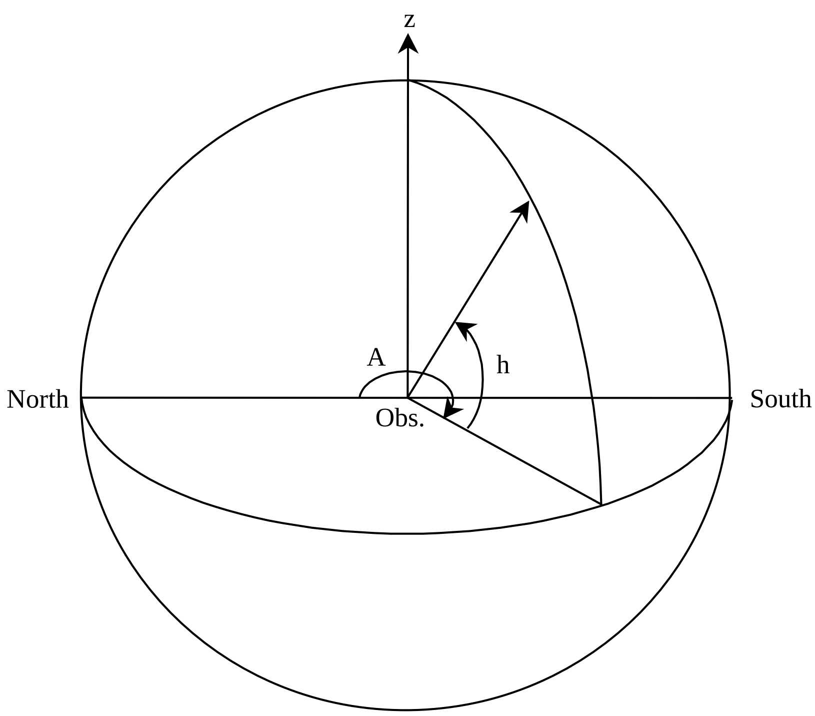

Figure 1

The hour angle - declination coordinate system:

This coordinate system is linked to the center of the geoid and is defined in the following way:

- The Astronomical Meridian is a vertical plane which contains the axis joining the two Celestial Poles. The elevation of the North Pole is also the astronomical or geographic latitude of the point. The Celestial Equator has its center at the geocenter and is perpendicular to the axis joining the two Celestial Poles. The Astronomical Meridian contains the local normal at the observer's location. A point on this Celestial Sphere is defined by two angles:
- The hour angle, H, counted positive westwards from the local Astronomical Meridian.
- The declination, δ, counted positive towards the North Pole and negative towards the South Pole.

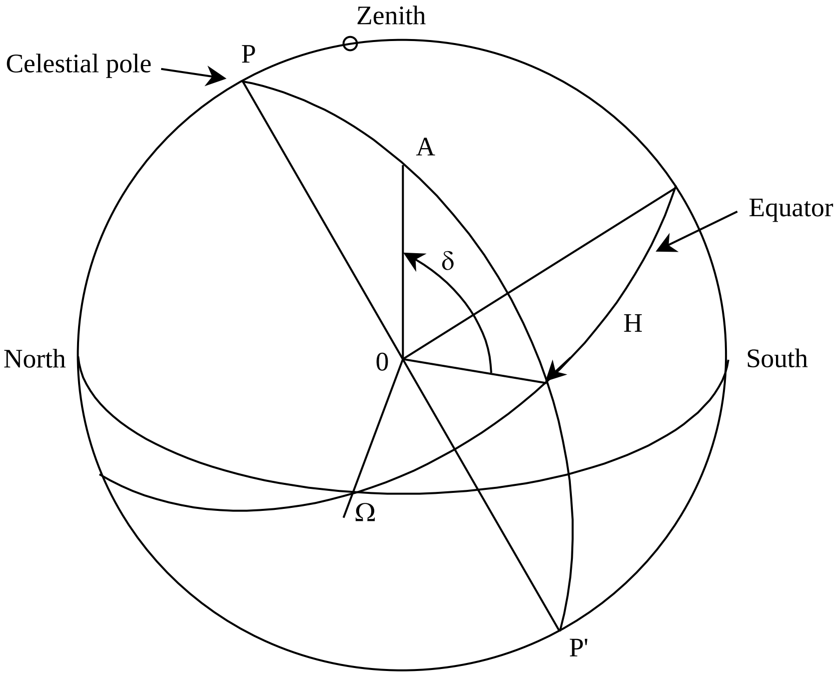

Figure 2

The right ascension - declination coordinate system:

This coordinate system has its center at the center of the geoid. The Equator is the Celestial Equator defined in the previous paragraph. The position of an object is defined by the declination defined in the previous paragraph. The other coordinate is the right ascension, α, counted positive eastwards from the Vernal Point γ. There is a relationship between the sideral time T, The hour angle H, and the right ascension α of any celestial object:

$$H = T - \alpha$$

When the celestial object is the Vernal Point γ, $\alpha = 0$. The sideral time is also the hour angle of γ.

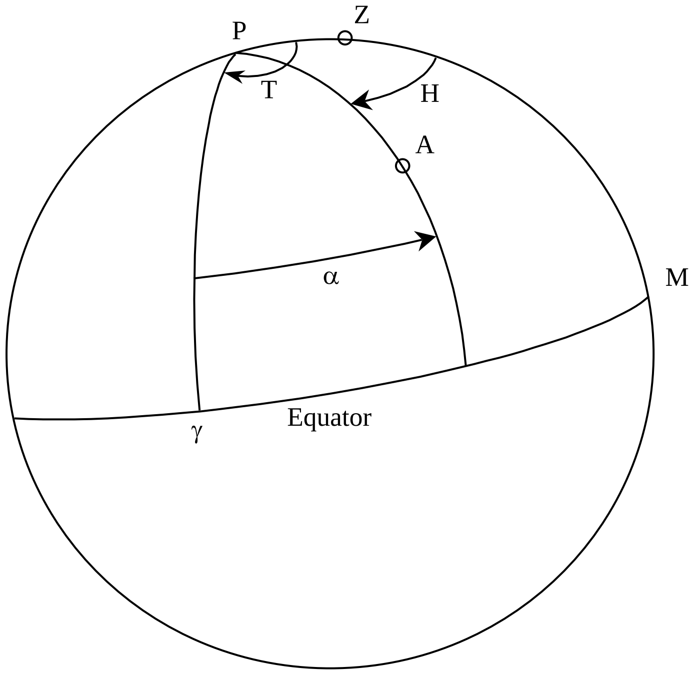

Figure 3

The right ascension with respect to the CNES Vernal Point 1950* is calculated in routine **tsidrg.** The right ascension with respect to the Mean Vernal Point* is calculated in routines **soltervo, solterv, solter00, solter05, solter10 and solter15.** These last routines differ only in the calculation of thetdip, phidip, which describe the tilt of the dipole from 1945 to 2015. The transformation of the coordinates and velocity components of a spacecraft from the inertial coordinate system to the geocentric coordinate system is performed in **pvig.**

<u>The Sun Position and Motion</u>:

In the Celestial system the Sun moves along the Ecliptic. The inclination of the Ecliptic versus the Equator is called Obliquity ε.

* See paragraph 8.3 for details

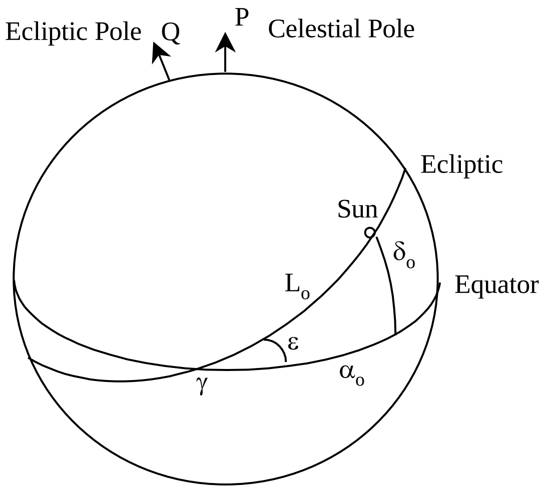

Figure 4

The Ecliptic and the Equator cross each other at the Vernal Point γ, ascending node of the Ecliptic. The position of the Sun along the Ecliptic is defined by its longitude Lo counted from the Vernal Point. It is possible to calculate the right ascension and declination of the Sun by solving the spherical triangle:

$$\sin\delta_o = \sin\varepsilon \sin L_o$$
$$\sin\alpha_o = \cot\varepsilon \tan\delta_o$$
$$\cos\alpha_o = \cos L_o / \cos\delta_o$$

The calculation of the Ecliptic longitude as well as the right ascension and the declination of the Sun are given by Russel (Ref. 1). The right ascension and the declination of the **sun** are calculated in routine **sun.** The obliquity is also calculated in routine **sun.** These results can also be found in routines **solter15, solter10, solter05, solter00, solterv, soltervo.**

8.3 FRAME OF REFERENCES REVISITED (IT WAS TOO SIMPLE!)

As mentioned above the origin of the right ascensions is the Vernal Point and this Vernal Point is defined as the intersection of the Celestial Equator and of the Ecliptic. Unfortunately, the Earth is submitted in its motion around the Sun to the gravitational influences of the Sun and the Moon as well as the other planets. The Sun and the Moon act on this ellipsoidal shaped body and the resulting attraction creates a torque: the Pole of the Earth is subject to the Precession and Nutation. On a very small scale the Pole of the Earth has an erratic motion due to the tides. All these perturbations change the location of the Pole, thus the location of the Vernal Point. It is therefore necessary to define very clearly these different effects.

<u>Motion of the Pole of the Earth</u>:

The true Earth's frame of reference OXYZ is defined in the following way:

Origin O at the barycenter of the Earth
Z axis in the direction of the true Pole Pv (axis of the instantaneous rotation of the Earth)
X axis in the true equator plane, the plane XZ contains the origin of the longitudes Go (convention)

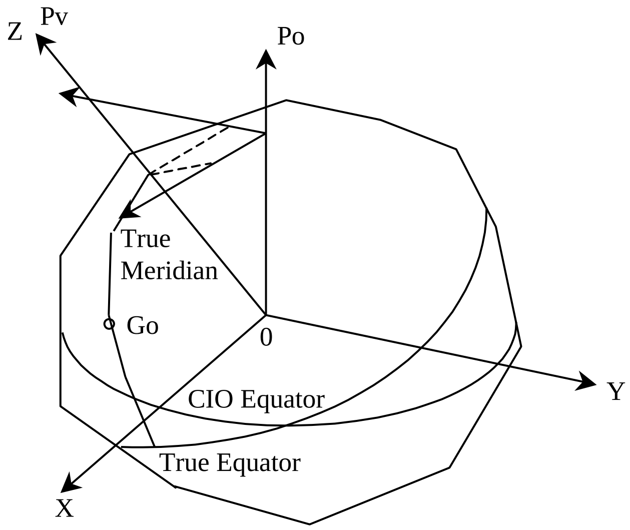

Figure 5

The location of Pv changes with time and its location is defined by its coordinates in an arbitrarily fixed frame called CIO-BIH.

<u>The Motion of the Ecliptic</u>:

The Ecliptic is defined as the intersection of the Celestial Sphere with the orbital plane of the Earth around the Sun.
The Ecliptic plane has a secular motion and at a given time the Ecliptic plane E(t) is defined with respect to the reference Ecliptic E(o) through some angle d.

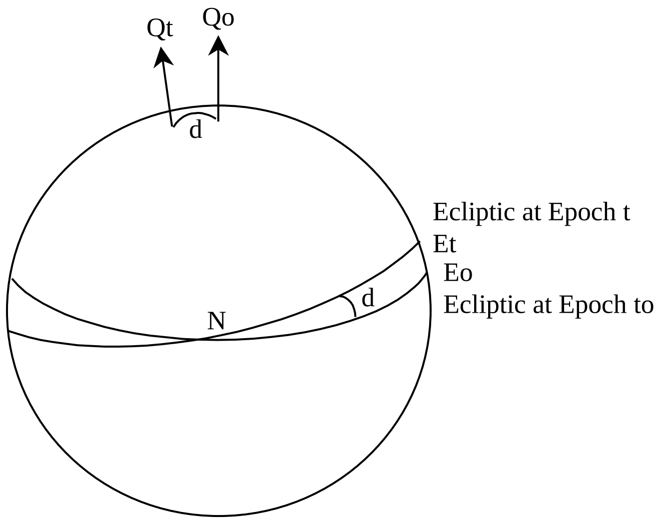

Figure 6

<u>Precession and Nutation of the Earth</u>:

The motion of the Earth around its barycenter is subject to the perturbations caused by the Moon and the Sun. These perturbations have two effects:

- A secular motion called Precession (26000 years period)
- A short periodic oscillation called Nutation (18.6 years period)

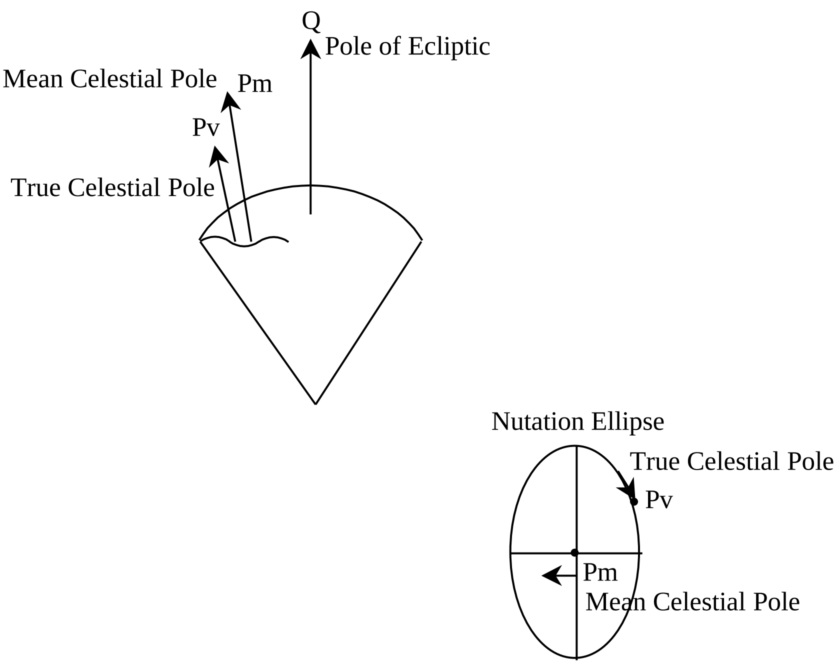

Figure 7

The Mean Pole motion is subject to Precession only: It is possible to define a Mean Celestial frame of reference with axis Z through the Mean Pole Pm and X axis at the intersection of the Mean Equator and the Ecliptic. Star catalogue information is given in the Mean Celestial frame of reference. The intersection of the Mean Equator with the Ecliptic defines on the Celestial Sphere the Mean Vernal Point.

The true Pole Pv of the Earth results from the Precession and the Nutation effects. The true Celestial frame of reference has axis Z through the true Pole Pv and the axis X is the intersection of the true Equator and the Ecliptic. The intersection of the true Equator and the Ecliptic defines on the Celestial Sphere the true Vernal Point.

CNES introduces a Modified Vernal Point which is the projection of the Mean Vernal Point 1950 on the true Equator of date.

For the Cluster project, the orbital information is given in a mean equatorial system of Epoch 2000,0. For Epoch 2000,0 the Vernal Point 2000,0 is at a given location intersection of Mean Equator for 2000,0 and Ecliptic for 2000,0. For another Epoch, 2001,0 for example, it is necessary to calculate the location of the Vernal Point 2001,0 and we take into account the Precession between these two epochs.
For Interball we transform the orbital elements calculated by the Russian Space Agency and given in a mean equatorial system 2000 into the CNES true Equator system. In this transformation we take into account the Precession and the Nutation effects. The introduction to the Precession and Nutation calculations involves the knowledge of the Astronomical Time References.

8.4 THE ASTRONOMICAL TIME REFERENCES

<u>The Julian Date</u>:

The Julian date defines a decimal date counted since the 1st January 4713 B.C. Julian Days start at 12 h U.T.
The Julian Day 0,1900 corresponds to the 31st of December 1899 at 12h U.T. Its Julian Date is 2415020.0 and Julian Day January 0,1950 corresponds to Julian Date 2433282 (31st of December 1949, 12 U.T.)

<u>The Modified Julian Date</u>:

The origin is November the 17th 1858 at 0h U.T. The relationship between the Modified Julian Date and The Julian Date is

MJD = JD - 2400000.5

For January 1st 1950 at 0h U.T. the modified Julian date is

MJD1950 = 2433282.5 - 2400000.5
MJD1950 = 33282.0

The CNES Julian Date:

The CNES Julian Date JUL is counted from January 1st 1950 at 0h UT
For January 1st 1950 0h UT JUL = 0.
For any other date we have

JUL = MJD - MJD1950 = JD - 2433282.5

For January 1st 2000, 0h U.T., JUL = 18262
The CNES julian Date is calculated in routine **julg.** It is always necessary to add the fraction of time between the hour, min, sec of the date and 0h0min0sec. For example the julian date for
January, 2, 1990 is: 14611., the Julian date for January 2, 1990 at 18h15min 6sec is simply

14611 + (18x3600 + 15 x 60 + 6) /86400. = 14611.760486

Routine **calendg** transforms an integer Julian date into a Gregorian date (year, month, day). **djgreg** transforms a Julian date and its fraction into a Gregorian date (year, month, day, hour, minutes, seconds). A useful routine **datjhms** transforms an interval in seconds into days, hours, minutes, seconds.

Epoch J2000:

Epoch J2000 is defined as January 1st, 2000 at 12h U.T.

J2000 corresponds to Julian date JD = 2451545.0

The Modified Julian Date 2000 of ESOC:

For the Cluster Project, ESOC introduces the Modified Julian Date 2000, MJD2000. It is chosen to be zero for January 1st, 2000, at 0h U.T. For dates prior than January 1st 2000 this Modified Julian Date is negative.

MJD2000 = MJD1950 - 18262 = JD - 2451544.5

Routine **jd2000** transforms a gregorian date into the ESOC julian date with respect to the reference epoch 2000.

8.5 THE CALCULATION OF THE PRECESSION AND THE NUTATION

The precession matrix:

The precession matrix converts a vector in the mean geocentric equatorial system of 2000.0 to the mean geocentric equatorial system of date. (The Mean Pole Pm moves from Pm(2000) to Pm(t)).

This precession matrix is the products of three elementary rotations:

- 1rst rotation around axis Z0 by an amount $-\zeta_a$ gives new axes X1,Y1,Z1
- 2nd rotation around axis Y1 by an amount θ_a gives axes X2,Y2,Z2
- 3rd rotation around axis Z2 by an amount $-z_a$ gives axes X3,Y3,Z3

The product of the three matrices gives the final matrix:

$$P=\begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}$$

with:

$$\begin{aligned}
P_{11} &= \cos Z_A \cos\theta_A \cos\zeta_A - \sin Z_A \sin\zeta_A \\
P_{12} &= -\cos Z_A \cos\theta_A \sin\zeta_A - \sin Z_A \cos\zeta_A \\
P_{13} &= -\cos Z_A \sin\theta_A \\
P_{21} &= \sin Z_A \cos\theta_A \cos\zeta_A + \cos Z_A \sin\zeta_A \\
P_{22} &= -\sin Z_A \cos\theta_A \sin\zeta_A + \cos Z_A \cos\zeta_A \\
P_{23} &= -\sin Z_A \sin\theta_A \\
P_{31} &= \sin\theta_A \cos\zeta_A \\
P_{32} &= -\sin\theta_A \sin\zeta_A \\
P_{33} &= \cos\theta_A
\end{aligned}$$

The angles ξ_A, θ_A, Z_A are defined as:

$$\begin{aligned}
\zeta_A &= 2306''.2181 T + 0''.30188 T^2 + 0''.017998 T^3 \\
\theta_A &= 2004''.3109 T + 0''.42665 T^2 - 0''.041833 T^3 \\
Z_A &= 2306''.2181 T + 1''.09468 T^2 + 0''.018203 T^3
\end{aligned}$$

Where the angles have to be converted in radians. T is the time interval expressed in Julian Centuries since epoch J2000.

If JD is the Julian date, the time interval in centuries is:

$$T = \frac{JD - 2451545.}{36525.}$$

Using the Modified Julian Date MJD2000 of ESOC T is also expressed as:

$$T = \frac{MJD2000 - 0.5}{36525}$$

Since there is a difference of 0.5 days between Epoch 2000 and MJD2000.

The precession matrix which transforms a vector in the inertial coordinate system J2000 into a vector in an inertial coordinate system of another Epoch is calculated in routine **pr2000.**

The Nutation:

The transformation of the coordinates of a point given in the Mean Equator of date, to the coordinates in the true Equator of date involves three rotations. A rotation ε_1 from the Mean Equator to the Mean Ecliptic of date, a rotation from the Mean Vernal Point to the true Vernal Point $\gamma_m \gamma_v = \Delta\Psi$ and a rotation from the Mean Ecliptic to the true Equator ε.

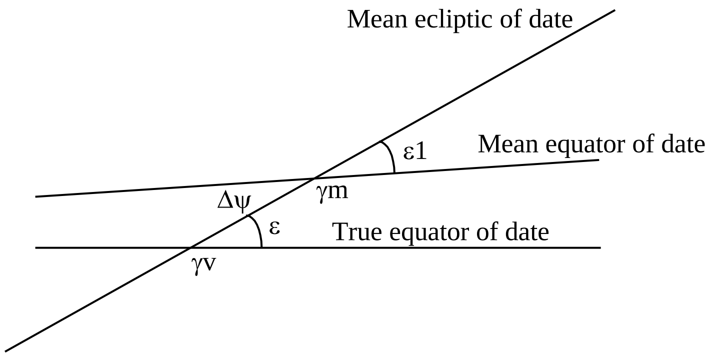

Figure 8

It is possible to choose between the theory of Newcomb or the theory of Lieske.
The nutation has rather small effects with a period of 18.6 years. The true Pole describes an ellipse around the Mean Pole and at the same time the Mean Pole drifts due to the precession. As a consequence the true Pole describes a winding curve around the Ecliptic Pole. The true Vernal Point moves around the Mean Vernal Point and the true Celestial Equator oscillates around the Mean Celestial Equator. The motion of the true Vernal Point (Nutation in Longitude N) is given by:

$$\gamma_m \gamma_v = N = -17''.23 \sin\theta$$

The inclination of the true Equator with respect to the Mean Equator changes with time (Nutation in obliquity):

$$\Omega = \varepsilon_v - \varepsilon_m = 9''.21 \cos\theta$$

Where $\theta = 259°8' - 69629''\,(t - 1900{,}0)$

time t is expressed in tropical years. The tropical year is the interval of time which corresponds to a change of 360° of the mean longitude of the Sun.
ε_m and ε_V are the mean and true obliquities.

Transformation of a vector from the Mean reference frame J2000 to the Mean reference frame of another Epoch or to the true reference frame of another Epoch:

The Mean reference frame J2000 is defined as $OXYZ_{2000}$with axis OZ_{2000} through the Pole Pm_{2000} and axis OX_{2000} through the Vernal Point γm_{2000}. This Vernal Point is defined as the intersection of the Ecliptic2000 and the Mean equator 2000. For another Epoch t the axis OZ_t crosses Pole Pm_t and axis OX_t is the intersection of the Ecliptic and the Mean Equator at Epoch t. The transformation between these two reference frames is the Precession Matrix from Epoch 2000 to Epoch t.
If we want to transform from the Mean reference frame at Epoch 2000 to the true reference frame at Epoch t, we first perform the transformation from the Mean reference frame at Epoch 2000 to the Mean reference frame at Epoch t, then we apply the Nutation transformation from Mean reference frame at Epoch t to the true reference frame at Epoch t. For Cluster the second transformation is neglected.

8.6 THE DIFFERENT SIDERAL TIMES

It is possible to define the true sideral time T_V which corresponds to the true Vernal Point and the Mean sideral time T_m which corresponds to the Mean Vernal Point. The difference between the two sideral times corresponds to the angular separation between the two Vernal Points caused by the Nutation and projected into the equatorial plane:

$$T_m = T_v - N\cos\varepsilon$$

The formula for the mean sideral time is:

$$T_{m,0} = 99°.6909833 + 36000°.7689\ C + 0°.0001525 \qquad (1)$$

where $$C = \frac{JD - 2415020.0}{36525}$$

C are the Julian centuries. Each Julian century has 36525 Julian days.

2415020.0 corresponds to December 31st 1899.
Formula (1) corresponds to the mean sideral time of Greenwich at 0h UT. At any other time of the day t, the Greenwich sideral time is:

$$T_m = T_{m,0} + \dot{\theta} t$$

where t is the Universal Time UT and $\dot{\theta}$ is the angular velocity of the Earth expressed in degrees per solar minute.

$$\dot{\theta} = 0.25068447° / mn$$

The mean sideral time defined by Russell (Ref. 1) is the same but expressed differently:

$$T_m = \quad 99°.690983 + 0.9856473354 \text{ x DJ} + 360° \text{ x FDAY}$$

Where FDAY = Number of seconds in the day divided by 86400
And DJ = 365 x (IYEAR - 1900) + (IYEAR - 1901) / 4 + IDAY - FDAY - 0.5

Where IYEAR is the year number
IDAY is the day number in the year

This formula is valid between 1901 and 2099. This formula is calculated in routines **sun, solter15, solter10, solter05, solter00, solterv, soltervo.**

CNES defines the sideral time with respect to the CNES Vernal Point and uses the CNES Julian Date:

$$T_{mcnes} = \; 100°.075542 + 360°.985647348 \text{ x } JUL + 0°.2900 \text{ x } 10^{-12} \text{ x } JUL^2$$

Where *JUL* is the CNES Julian Date counted from 1st January 1950, 0h U.T. The CNES sideral time is calculated in routine **tsidrg.** The CNES Julian date is calculated in routine **julg.**

8.7 COORDINATE TRANSFORMATIONS FOR AN OBLATE EARTH

<ins>Transformation from geodetic coordinates to geocentric coordinates</ins>:

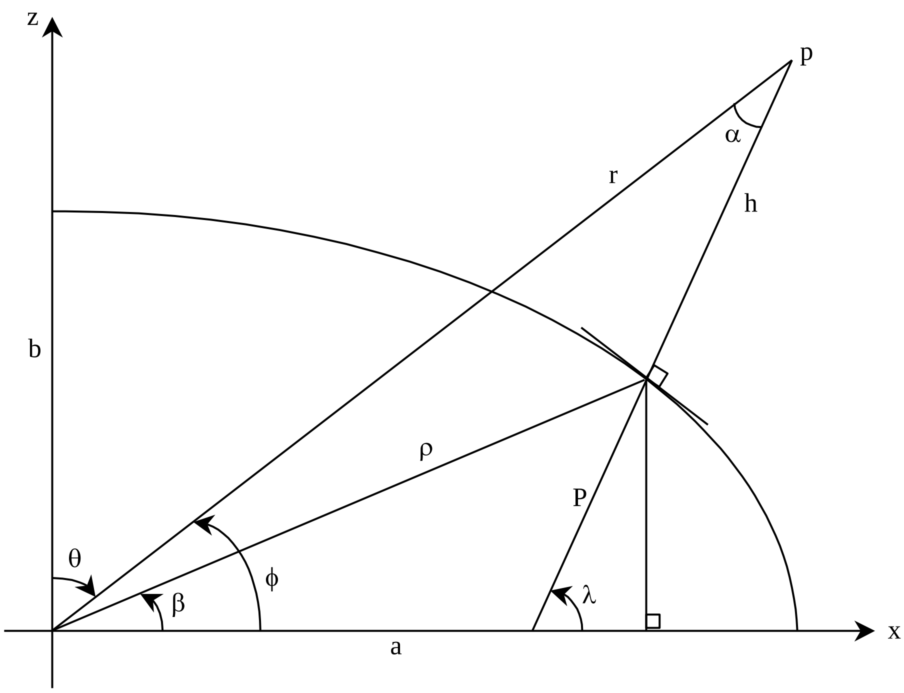

Figure 9

From the above figure we can calculate the geocentric coordinates. If a and b are the semi-major and semi-minor axes of the Earth's ellipsoid, the equation for the ellipsis is:

$$\frac{x^2}{a^2}+\frac{z^2}{b^2}=1$$

From the gradient we derive the components of the normal to the ellipsoid surface:

$$\vec{\nabla}=\frac{2x}{a^2N}\hat{i}+\frac{2z}{b^2N}\hat{k}$$

Where $N=\left\|\vec{\nabla}\right\|$

We also have $\tan\lambda=\frac{\nabla z}{\nabla x}=\frac{z}{x}\frac{a^2}{b^2}$

as $z=\rho\sin\beta$, $x=\rho\cos\beta$ $\tan\lambda=\frac{a^2}{b^2}\tan\beta$

Expressing $\sin\beta$ as a function of $\tan\beta$ we obtain $\sin\beta=\frac{\sin\lambda}{\left(\sin^2\lambda+\frac{a^4}{b^4}\cos^2\lambda\right)^{\frac{1}{2}}}$

we also have: $\rho^2 = a^2\cos^2\beta + b^2\sin^2\beta$

but $b^2 = a^2(1\text{-}e^2)$

We get :

$$\rho=a\sqrt{1-e^2\sin^2\beta}$$

The rectangular coordinates of the point P are:

$x = \rho\cos\beta + h\cos\lambda$
$x = \rho\sin\beta + h\sin\lambda$

It is easy to obtain the radial distance *r* and the geocentric colatitude θ of the point P.
The transformation from geodetic coordinates into geocentric coordinates is performed in routine **gdvgc.**

Transformation from geocentric coordinates to geodetic coordinates:

This transformation formula has been developed by J. Morrison and S. Pines (Ref. 2).

$r \cos \varphi = (h + c) \cos \lambda$
$r \sin \varphi = (h + s) \sin \lambda$

where $c = \left(1 - e^2 \sin^2 \lambda\right)^{\frac{1}{2}}$ and $s = (1 - e^2)\, c$

The couple of equations can be solved:

$$\tan \lambda = \tan \varphi + \frac{e^2}{\rho \cos \varphi} \frac{\sin \lambda}{\left(1 - e^2 \sin^2 \lambda\right)^{1/2}}$$

The solution of this equation is obtained by the Lagrange expansion formula. The geodetic latitude is given by:

$$\lambda = \varphi + a_2 (e, \rho) \sin 2\varphi + a_4 (e, \rho) \sin 4\varphi + a_6 (e, \rho) \sin 6\varphi + a_8 (e, \rho) \sin 8\varphi$$

$$a_2 = \frac{1}{1024\rho}\left(512e^2 + 128e^4 + 60e^6 + 35e^8\right) + \frac{1}{32\rho^2}\left(e^6 + e^8\right) - \frac{3}{256\rho^3}\left(4e^6 + 3e^8\right)$$
$$a_4 = -\frac{1}{1024\rho}\left(64e^4 + 48e^6 + 35e^8\right) + \frac{1}{16\rho^2}\left(4e^4 + 2e^6 + e^8\right) + \frac{15e^8}{256\rho^3} - \frac{e^8}{16\rho^4}$$
$$a_6 = \frac{3}{1024\rho}\left(4e^6 + 5e^8\right) - \frac{3}{32\rho^2}\left(e^6 + e^8\right) + \frac{35}{768\rho^3}\left(4e^6 + 3e^8\right)$$
$$a_8 = \frac{e^8}{2048}\left(-\frac{5}{\rho} + \frac{64}{\rho^2} - \frac{252}{\rho^3} + \frac{320}{\rho^4}\right)$$

where $e^2 = 2\varepsilon - \varepsilon^2$
and ε is the flattening of the Earth. The transformation from geocentric coordinates into geodetic coordinates is performed in routine **gcvgd**.

8.8 A SIMPLIFIED APPROACH TO THE DEFINITION OF THE SIDERAL TIME

In this paragraph we give a simplified formula for the definition of the sideral time which can be useful for a first approach in mission analysis.
According to A. Danjon (Ref. 3) the sideral time is the hour angle of the Vernal Point γ. We don’t know a priori where is located γ. However we know where is located the Sun with respect to Greenwich through the Universal Time UT and we also know the location of the Sun for some well defined epochs of the year.

At 12h UT the Sun is in the Greenwich Meridian and the 21st of March the Sun is at the Vernal Meridian γ. As the hour angle is counted from Greenwich, the 21st of March at 12h U.T. the Greenwich sideral time equals the hour angle and is 0.:

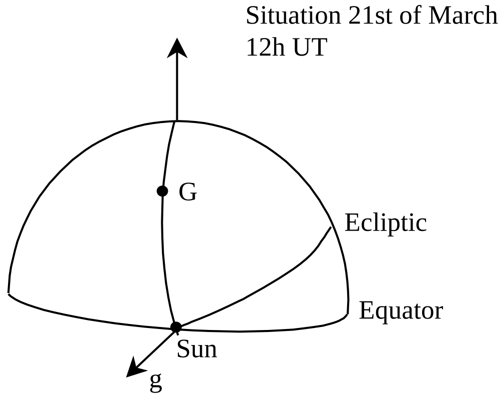

Figure 10

If J is the day of the year, the 21st of March corresponds to J = 80; Thus for 12h UT we have:

sideral time at 12h UT = -80 + J

For the 21st of June, the 21st of September and the 21st of December at 12h U.T. the Sun and the Greenwich Meridian are located at points 1,2,3:
Using the previous formula the sideral time is respectively 90°, 180°, 270° for points 1, 2, 3 for 12h U.T.
For a Universal Time different from 12h U.T., the Greenwich Meridian has rotated eastwards and the angle of rotation α is simply:

α = (UT – 12) x 15°

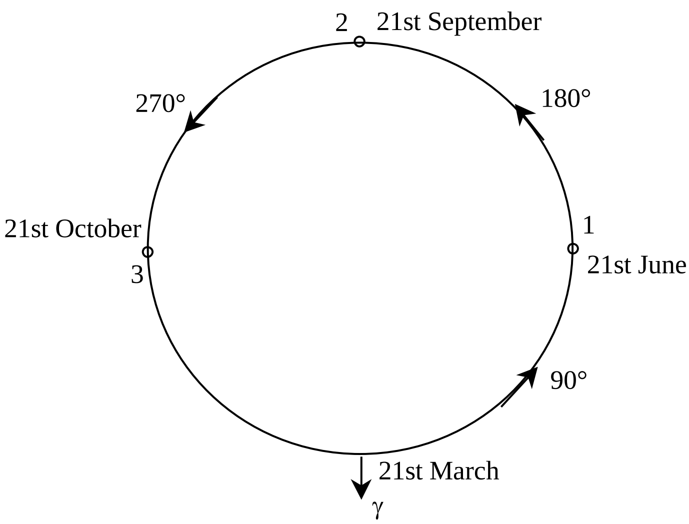

Figure 11

We thus have the general formula:

Greenwich sideral time at UT = sideral time at 12h UT + (UT-12) x 15°

Taking into account the definition of the sideral time at 12h U.T. we get:

Greenwich sideral time day J at UT = 100° + J + UT x 15°

With this formula, it is therefore possible to locate the Greenwich Meridian with respect to the Vernal Point, knowing the day of the year and the UT.
For the 1st of March at 10h U.T., we have J = 60, UT = 10. We get

Greenwich sideral time = 100° + 60° + 150° = 310°

The above result is approximate as well as the above formula but it helps finding the astronomical situation and locate Greenwich with respect to the Vernal Point.

8.9 CELESTIAL MECHANICS

<u>Orbital calculations:</u>

Orbital calculations of highly eccentric orbits require precise integration methods such as the Runge Kutta methods developed by Fehlberg or Dormand-Price. In these methods an upper limit of the error is imposed and the iteration process continues until the limit is reached. This iteration process is done for each integration step. As a consequence the orbit extrapolation is time consuming. Moreover the computer code must be provided to all users and increases the risk of unproper use. ESOC has developed an elegant method for orbital calculations which does not require much computer code for non expert users. This method is not time consuming and can be summarized as follows:

- The orbit determination and calculation is initially made in ESOC with all the necessary tools and methods in order to achieve the best precision.
- Each orbit is divided in a finite number of intervals.
- For each interval the average keplerian elements are calculated.
- A fit of the precise orbit is made using the keplerian orbit and Tchebycheff polynomials.

As a consequence the final user needs to have only the final keplerian elements for each interval and the associated Tchebycheff polynomials. The reconstruction is made by one routine called **orbit**. This routine reads a file provided by ESOC the keplerian elements and the Tchebycheff polynomials and reconstructs for a given time the exact position and velocity of the spacecraft.
ESOC provides the orbital information in an inertial frame of reference Epoch 2000. For Epoch 2000 the Vernal Point is defined as the intersection of the Mean Equator and the Ecliptic for this Epoch. As mentioned in the previous paragraphs it is necessary to take into account the precession between Epoch 2000 and the current Epoch. The precession matrix provided by ESOC, routine **pr2000**, transforms a position-velocity state vector for Epoch 2000 in a position-velocity state vector for the current Epoch. The routine **posin** calculates at a given time the position-velocity state vector of a Cluster spacecraft using successively the **orbit** and **pr2000** routines.

REFERENCES

Ref. 1 C. Russel: Geophysical Coordinates Transformations, Cosmic Electrodynamics 1971, 2, 184-196, D. Reidel Publishing Company

Ref. 2 J. Morrison and S. Pines: The Reduction from geocentric to Geodetic Coordinates, Astronom. J, 1961,66,1,15-16

Ref. 3 A. Danjon: Astronomie Generale, 1959, Sennac Editor, Paris

Ref. 4 R.M. Green: Spherical Astronomy, 1985, Cambridge University Press, Great Britain

9. MATHEMATICS

9.1 INTRODUCTION

In this section we describe the routines encountered in the transformation of vectors, conversion of angles,......

9.2 ANGLE OF A VECTOR WITH RESPECT TO AXIS X IN A (X, Y) COORDINATE SYSTEM KNOWING ITS TWO COMPONENTS X AND Y

Given two components *x*, *y* of a vector it is possible to calculate the angle between 0, 2π using fortran function a tan 2:

$$\alpha = a\tan 2(y, x) \tag{1}$$

This calculation is performed in routine **angleg**. For *x*, *y* both zero, the routine returns $\alpha = 0$ and in other cases α is always positive.

9.3 TRANSFORMATION OF THE CARTESIAN COORDINATES OF A POINT INTO ITS SPHERICAL COORDINATES

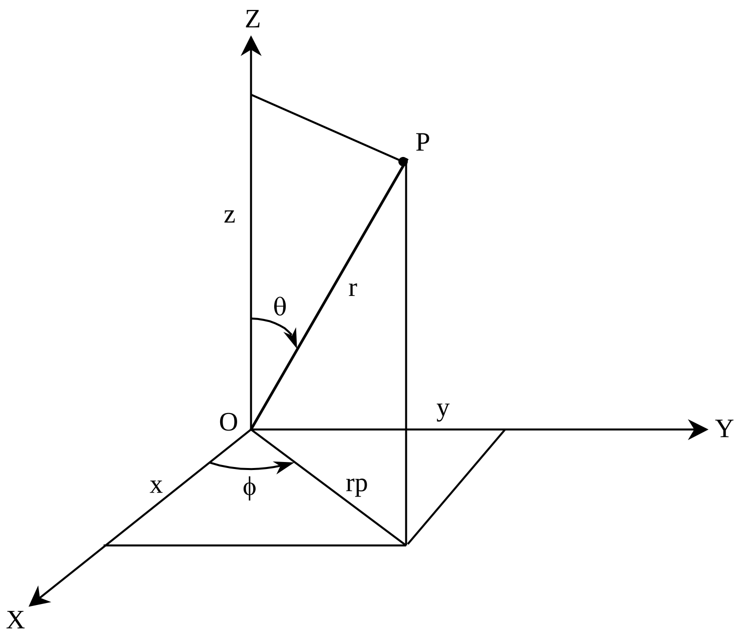

Figure 1

The spherical coordinates (r, θ, φ) are obtained from the cartesian coordinates (x, y, z) using the following formulae:

$$
\begin{aligned}
r &= \sqrt{x^2 + y^2 + z^2} \\
\theta &= a\cos\left(z/r\right) \\
\varphi &= a\tan 2\left(y, x\right)
\end{aligned} \tag{2}
$$

θ is the colatitude and φ is the azimuth.

We define $r_p^2 = \sqrt{x^2 + y^2}$. If $r_p \neq 0$ the above formulae can be applied.

If $r_p = 0$ we set $\varphi = 0$ and we have two possibilities:

$z \geq 0$ then $\theta = 0$
$z < 0$ then $\theta = \pi$

This transformation is performed in routine **carsp**.

9.4 TRANSFORMATION OF THE SPHERICAL COORDINATES OF A POINT INTO ITS CARTESIAN COORDINATES

The transformation from spherical coordinates to cartesian coordinates is defined as:

$$
\begin{aligned}
x &= r \sin\theta \cos\varphi \\
y &= r \sin\theta \sin\varphi \\
z &= r \cos\theta
\end{aligned} \tag{3}
$$

where θ is the colatitude of the point and φ is the azimuth. r is the radial distance to the origin. This transformation is performed in routine **spcar**.

9.5 PRODUCT OF A COLUMN MATRIX BY A RECTANGULAR (UNITARY) MATRIX

The product is defined by the formula:

$$
y_i = \sum_{j=1}^{3} a_{ij}\, x_j \tag{4}
$$

The matrix a_{ij} is a matrix of dimensions (3,3), vectors x, y have three components (x_1, y_1, z_1) and (y_1, y_2, y_3).

This calculation is performed in the routine **promal**.

9.6 PRODUCT OF TWO UNITARY MATRICES

The product of two matrices is defined as:

$$c_{ij} = \sum_{k=1}^{3} a_i^k \, b_k^j \qquad (5)$$

The transpose (inverse) matrix is defined as $\bar{c}_i^{\,j} = c_j^i$

The product of two matrices of dimension (3,3) is performed in routine **promat** which also provides the inverse matrix of the result.

9.7 TRANSFORMATION OF THE RECTANGULAR COMPONENTS OF A VECTOR INTO SPHERICAL COMPONENTS

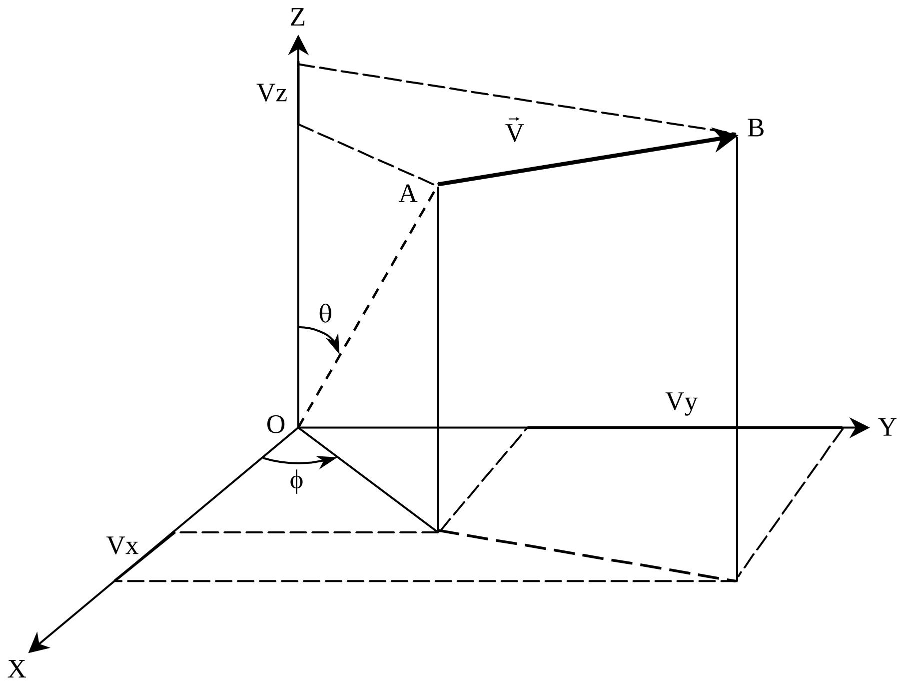

Figure 2

If point A is located in (r, θ, φ) we get:

$$\begin{aligned} V_r &= (V_x \cos\varphi + V_y \sin\varphi)\sin\theta + V_z \cos\theta \\ V_\theta &= (V_x \cos\varphi + V_y \sin\varphi)\cos\theta - V_z \sin\theta \\ V_\varphi &= V_y \cos\varphi - V_x \sin\varphi \end{aligned} \qquad (6)$$

This transformation is performed in routine **vcarvsp**.

9.8 TRANSFORMATION OF THE SPHERICAL COMPONENTS OF A VECTOR INTO RECTANGULAR COMPONENTS

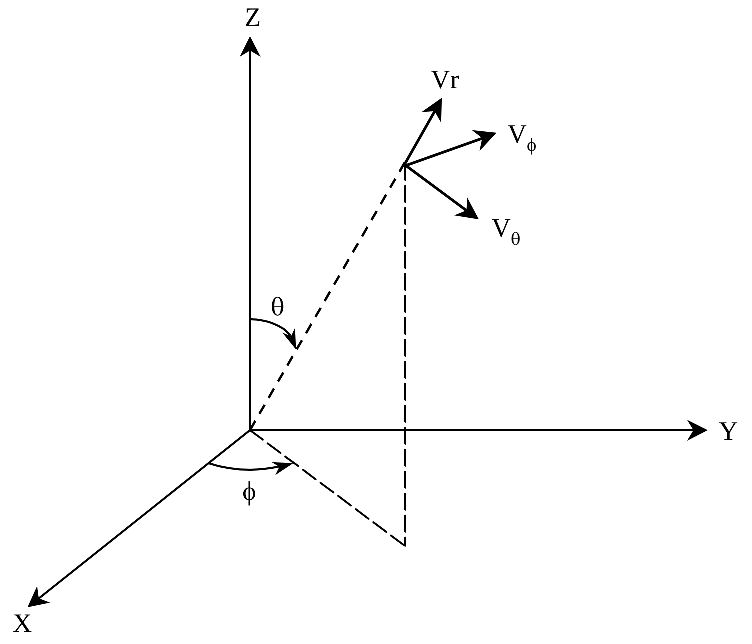

Figure 3

We get:

$$\begin{aligned} V_x &= \left(V_r \sin\theta + V_\theta \cos\theta\right)\cos\varphi - V_\varphi \sin\varphi \\ V_y &= \left(V_r \sin\theta + V_\theta \cos\theta\right)\sin\varphi + V_\varphi \cos\varphi \\ V_z &= V_r \cos\theta - V_\theta \sin\theta \end{aligned} \quad (7)$$

The transformation is performed in routine **vspvcar**.

9.9 LAGRANGE INTERPOLATION FORMULA

Interpolation can be performed by a Lagrange polynomial of order 2. Using the definition of Abramowitz and Stegun (Ref. 1) we have:

$$f(x) = \sum_{i=0}^{n} l_i(x)\, f_i \quad (8)$$

where $$l_i(x) = \frac{\Pi_n(x)}{(x - x_i)\,\Pi_n'(x_i)}$$

$l_i(x)$ is of the form: $$\frac{(x - x_0)\cdots\cdots(x - x_{i-1})(x - x_{i+1})\cdots\cdots\cdots\cdots(x - x_n)}{(x_i - x_0)\cdots\cdots(x - x_{i-1})(x_i - x_{i+1})\cdots\cdots\cdots\cdots(x_i - x_n)}$$

We obtain for n = 2

$$l_0(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)}$$
$$l_1(x) = \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)}$$
$$l_2(x) = \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_0)}$$

and the interpolation formula for n = 3 is:

$$f(x) = l_0(x)\, f_0 + l_1(x)\, f_1 + l_2(x)\, f_2$$

where f_0, f_1, f_2 are the values of $f(x)$ at points x_0, x_1, x_2.
This formula has been used for the calculation of the conjugate point at a given altitude or at the surface of the Earth using three calculated points.

REFERENCES

Ref. 1 M. Abramowitz and I. A. Stegun : Handbook of mathematical functions, 1968, Dover, New York